1 Enhanced Charlson Comorbidity Index varies more among older people in

- 2 four U.S. health systems
- 3 (Title has 94 characters including spaces; same as Short title.)
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20 Abstract

21 The enhanced Charlson Comorbidity Index (eCCI) combines a person's concurrent chronic 22 medical conditions (comorbidities) in 39 categories into an index. We show that the average 23 eCCI and the variance of eCCI of people of each age increase with age in de-identified electronic 24 health records of 238,156 adults in low-income communities served by four health systems (2 in 25 Chicago, 2 in NYC). The variance of eCCI approximates a power function of the mean eCCI. 26 This quantitative relationship, not previously recognized, approximates Taylor's power law of 27 fluctuation scaling. The quintiles of eCCI diverge with increasing age, consistent with an increasing variance. Within almost every age group, the frequency distribution of eCCI per 28 29 individual is well approximated by a negative binomial distribution. This frequency distribution 30 of eCCI per individual of a given age can account approximately for the relation of variance to 31 mean across ages. The increase with age in the mean and the variance of eCCI, and the faster 32 increase of the variance compared to the mean, show that aging brings more comorbidity on 33 average and greater differences in comorbidity among individuals of more advanced ages. 34 Clinical practice should recognize and respond to the greater differences in comorbidity among 35 older people.

37 Introduction

According to the "World Report on Ageing and Health" (Beard et al. 2016, p. 2146), "ageing is 38 39 ... associated with an increased risk of a person having more than one disorder at the same time 40 (multimorbidity)." Moreover, "the older the age group, the greater the variation found in cognition, physical and sensory function, and social engagement" (Santoni et al. 2015, p. 2). 41 42 Hultsch et al. (2002) distinguished three types of variability in the reaction time of younger and 43 older adults: "variability between persons (diversity), variability within persons across tasks 44 (dispersion), and variability within persons across time (inconsistency)." In the present cross-45 sectional study, each individual is assessed once only by chronological age and an enhanced 46 Charlson Comorbidity Index (eCCI), an indicator of multimorbidity. This structure of the data 47 gives no opportunity to measure inconsistency (which would require longitudinal data) or 48 dispersion (which would require comparing different indices of multimorbidity). We analyze 49 only the mean, variance (Hultsch's "diversity"), quintiles, and frequency histogram of eCCI 50 within groups of people of the same chronological age. The quantitative relationship between the 51 average (or mean) and the variance of eCCI among adults as a function of age appears not to have been examined (e.g., Divo et al. 2014, Chen et al. 2022, Chen et al. 2023, Chowdhury et al. 52 53 2023, Bensken et al. 2024).

54 Before examining data, we predicted that the eCCI might obey a pattern widely observed in 55 ecology and known as Taylor's law (L. R. Taylor 1961, R. A. J. Taylor 2019). Taylor's law 56 predicts that the variance of eCCI should approximate some power of the mean of eCCI across 57 the entire range of adult ages. The data here partly confirm and partly reject this predicted 58 power-law relationship of variance to mean.

59 Data and definitions

- 60 Four health systems (HS 1, HS 2, HS 3, HS 4) in the United States measured the age and the
- 61 enhanced Charlson Comorbidity Index (eCCI) (Charlson et al. 2025), based on electronic health
- 62 records, of 238,156 adults in low-income communities. The eCCI used here was designed to
- 63 anticipate risk of hospitalization and disability whereas several earlier versions of CCI were
- 64 designed to anticipate risk of death (Charlson et al. 1994, Charlson et al. 2022, Charlson and
- 65 Wells 2022; see also Ly et al. 2025 and the on-line calculator at
- 66 <u>https://www.mdcalc.com/calc/3917/charlson-comorbidity-index-cci</u>). The eCCI of a patient was
- 67 the sum of the points assigned in Table 1 to each chronic condition in the patient's electronic
- 68 medical record. These 39 conditions generate $2^{39} = 549,755,813,888$ or almost 550 billion
- 69 possible combinations of conditions. The data analyzed here were accessed for research purposes
- 70 on 2024-12-28.
- 71

72 Table 1. Chronic conditions and assigned points in the enhanced Charlson Comorbidity Index for

73	Tipping Points Conditions and	Weights (Charlson et al., 2025).

Chronic Condition	Comorbidity Weight
Myocardial infarction	1
Congestive heart failure	1
Peripheral vascular disease or bypass	1
Cerebrovascular disease or transient ischemic disease	1
Hemiplegia	2
Pulmonary disease/asthma	1
Diabetes	1
Diabetes with end organ damage	2
Moderate to severe renal disease	2
Mild liver disease	2
Severe liver disease	3
Gastric or peptic ulcer	1
Cancer	2
Rheumatic or connective tissue disease	1
Hypertension	1
Skin ulcers/cellulitis	2
Depression	1
Warfarin	1
Inflammatory bowel disease	1
Sickle cell disease	3
Hemophilia	3
Muscular dystrophy	2
Cystic fibrosis	3
Tay Sachs	3
Cerebral palsy	2
Uncontrolled seizures	3
Dementia or Alzheimer's	1
Dialysis	2
Developmental delay	2
Mental retardation	2
Down's syndrome	3
Bipolar disease	3
Antipsychotics	3
Drug or alcohol addiction	3
Schizophrenia	3
Autism	3
Metastatic solid tumor	6
HIV or AIDS	6
Any transplant: Renal, heart, liver, bone marrow, or lung	6

74 Age (in whole years plus days from the date of birth to the date on which eCCI was computed) 75 was rounded to the nearest whole year. Single years of age with 100 or more people were 76 selected for further analysis because age groups with few people (here, only the oldest elderly) 77 were judged to give unreliable estimates of the mean and variance (Taylor et al., 1988). 78 Two of the four health systems (HS 1 = NYC-a and HS 2 = NYC-b) were in New York City and 79 two (HS 3 = Chicago-a and HS 4 = Chicago-b) were in Chicago, Illinois. In New York, the two 80 health systems were Family Health Centers at NYU Langone and Community Healthcare 81 Network. In Chicago, the two health systems were Erie Family Health Centers and Friend 82 Health. In neither city is it publicly known which health system was "a" and which was "b." The 83 health systems and the individuals were fully de-identified for this analysis.

84 **RESULTS**

85 Age structure by HS

86 The frequency histograms of the number of people by age in each HS (Figure 1) show visible

87 differences among the HSs in the modal age. The modal age of HS 2 is by far the oldest.

- Figure 1 Number of people of each age with at least 100 people, by health system. Source:
- 90 NumberOfPeopleEachAge4HSagemin100_20250131-122802.png







To examine whether the differences in age distribution between and among HSs could have been
due to sampling fluctuations from a common single underlying distribution, we used nonparametric and parametric tests.

96 The non-parametric Kruskal Wallis test (Matlab kruskalwallis) revealed statistically

97 significant differences in the ranks of the age of individuals in all four HSs. HS 2 had the oldest

98 age distribution, as Figure 1 suggests. Including only HSs 1, 3, 4, or only the pairs of HSs 1 and

99 3, or HSs 1 and 4, or HSs 3 and 4, the Kruskal Wallis test again revealed statistically significant

100 differences in the distributions of the age of individuals in different HSs. In all cases, p values

101 (testing the null hypothesis of no difference in age distribution) were shown as 0, meaning that

102 they fell below some small positive threshold for rounding to 0.

103 The parametric test of a difference in mean ages between HSs was analysis of variance (Matlab

104 anova1). The mean ages, shown in Table 2, are presumably asymptotically normally

105 distributed by the central limit theorem even if the overall age distribution is not normally

106 distributed. Once again, the p value was shown as 0. We again reject the null hypothesis that the 107 age distributions of the four HSs were sampled from the same underlying distribution.

108 Table 2 summarizes the distributions of ages and eCCI by HS. The mean eCCI and the variance

109 of eCCI in HS 3 and HS 4 (in Chicago) were notably smaller than the mean and variance of HS 1

110 and 2 (in New York), and HS 2 had notably higher mean age and mean eCCI than the three other

111 HSs. The Discussion offers possible explanations of this difference between cities.

112	Table 2. Summary	statistics of	age and eCCI ir	n four health systems.
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Health system \rightarrow	1 NYC-a	2 NYC-b	3 Chicago-a	4 Chicago-b
Aunoule J				
Number of people	39018	41400	91568	66170
Minimum age years	18	18	19	19
Median age years	39	52	39	40
Maximum age years	106	108	112	110
Mean age	42.3493	51.3300	41.6381	43.4239
Standard dev. age	15.8985	17.7623	15.5392	16.4802
Minimum eCCI	0	0	0	0
Median eCCI	1	2	0	0
Maximum eCCI	24	22	23	21
Mean eCCI	1.4322	2.4736	1.0860	0.8847
Standard dev. eCCI	2.2007	2.3270	1.7543	1.6422

113

114 Mean and variance of eCCI by age group

115 In each HS, for each age with 100 or more individuals, the mean eCCI and the variance of the

116 eCCI of the individuals of that age generally increase with age (Figure 2). For each age, the area

117 (not radius) of the circle for the variance of eCCI is proportional to the number of individuals of

that age. The smallest circles at the extreme right of each panel show ages with 100 individuals.

The mean eCCI by age is substantially higher in HS 1 and HS 2 (New York) than in HS 3 and HS 4 (Chicago), and the variance of eCCI is substantially higher in HS 1 and HS 2 than in HS 3 and HS 4. Figure 2 shows clearly that the Poisson distribution (where variance equals mean) could not adequately describe the frequency distribution of eCCI by age group in any HS.

Figure 2 The mean eCCI (blue circles) and the variance (red circles) of the eCCI of the individuals of each single year of age with 100 or more individuals, by HS. The area of each red circle is proportional to the number of individuals of the age corresponding to the abscissa of the center of the circle. Source: MeanCCIvsAgeVarCCIvsAge4HSagemin100_20250131-

128 122802.png



129

130

132 Variance function

133 The variance function of eCCI in a set of ages shows the variance of eCCI at each age group on

the vertical axis as a function of the mean eCCI at each age on the horizontal axis. For each HS,

135 Figure 3 displays the variance function of eCCI by age with at least 100 individuals (circles).

136 The area of each circle is proportional to the number of individuals in the age group. The color of

137 each circle shows the age of the individuals in the group. Figure 3 also displays two models for

138 comparison with the eCCI data: Taylor's law and the quadratic Taylor's law.

139 Taylor's power law of fluctuation scaling

140 Taylor's law (TL) was discovered independently at least three times before L. R. Taylor (1961)

141 brought it to wide attention. To describe TL, let v(a) denote the sample variance of eCCI and

142 m(a) the sample mean of eCCI at age a. These quantities are functions of the data, hence the

143 qualification "sample." By contrast with the sample moments, let $\sigma^2(a)$ and $\mu(a)$ denote the

- population variance and the population mean, respectively, of eCCI in a model of the variancefunction
- 145 function.

146 Figure 3 The variance of the eCCI of the individuals of each age with 100 or more individuals as

147 a function of the mean eCCI of each group (center of the circles) and two models: Taylor's law

148 (3) (black solid straight line) and the quadratic Taylor's law (4) (blue dashed curved line). HS 1

and HS 2 are in New York. HS 3 and HS 4 are in Chicago. The area of each circle is proportional

150 to the number of individuals in the corresponding age. The color of each circle represents age

151 from youngest (deep purple or blue) to oldest (yellow). Source:

152 logVarCCIvslogMeanCCITLQTLall4HSagemin100_20250131-140652.png



\log_{10} variance of CCI versus \log_{10} mean of CCI in four health systems





TL proposes that the variance function is a power law, meaning that the population variance is proportional (with coefficient c) to some power (denoted b here) of the population mean for a range of values of age a:

158 (1)
$$\sigma^2(a) = c[\mu(a)]^b, c > 0$$
, for all ages *a*

and that the sample variance is approximately related to the sample mean by a relationship of thesame form with an unspecified error model:

161 (2)
$$v(a) \approx c[m(a)]^b, c > 0$$
, for all ages *a*.

We say that *c* is the coefficient and *b* is the exponent of TL. TL (2) implies a linear relationshipon log-log coordinates:

164 (3)
$$\log_{10} v(a) \approx \log_{10} c + b \log_{10} m(a).$$

165 The so-called quadratic Taylor's Law (Taylor et al. 1978, p. 388, Eq. (14)) (QTL) generalizes TL



167 (4)
$$\log_{10} v(a) = c_1 + b_1 \log_{10} m(a) + b_2 [\log_{10} m(a)]^2$$
,

168 In each HS, to estimate the values of c, b, c_1, b_1, b_2 from data for ages a with at least 100 individuals, we fitted linear (3) and quadratic (4) regressions by least squares using Matlab 169 170 function regress (black solid straight line and blue dashed curve in Figure 3). 171 Table 3 gives the estimated exponent b and estimated coefficient c and their 95% confidence 172 intervals (CIs). These confidence intervals are approximate because they assume the mean eCCI 173 is known without sampling error. In HS 1 in New York, the exponent b does not differ 174 significantly from 1, while b is significantly less than 1 in HS 3 in Chicago and is significantly greater than 1 in HS 3 and HS 4. The coefficient $c = 10^{\log_{10} c}$ significantly exceeds 1 in all four 175 176 HSs. 177

Table 3. Least squares estimates of the coefficient c and the exponent b of sample Taylor's law

179 (3) and nominal 95% confidence intervals based on each age's mean and variance of eCCI. For

180 example, in HS 1, with mean eCCI m(a) and variance v(a) of eCCI in age a, the best

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181 approximating TL is \log v(a) \approx 0.4171 + 1.0729 \cdot \log m(a) for all ages a.
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health	exponent_b	low_b	high_b	log10_c	low log10_c	high log10_c
system						
1 NYC-a	1.0729	0.95743	1.1884	0.4171	0.37991	0.45429
2 NYC-b	1.2287	1.0861	1.3713	0.15558	0.096415	0.21475
3 Chicago-a	0.92951	0.87851	0.98052	0.35842	0.3422	0.37464
4 Chicago-b	1.108	1.0442	1.1718	0.46305	0.45062	0.47548

- 183 Non-linearity of log variance as a function of log mean
- 184 In Figure 3, on log-log coordinates, the variance functions of HS 1, HS 2, and HS 3 are not well
- approximated as straight lines on log-log coordinates, contrary to TL. Rather, the variance
- 186 functions of HS 1 and HS 2 appear to be concave.
- 187 Table 4 gives the OLS estimates and 95% confidence intervals of the parameters b_2 , b_1 , c_1 in
- 188 order from left to right. The estimated exponents b_2 are significantly less than 0 because the 95%
- 189 confidence interval lies entirely to the left of 0. By contrast, b_2 for HS 4 does not differ
- 190 significantly from 0.
- 191 Table 5 gives the statistics of the TL and QTL regressions.

193 Table 4. Parameter estimates and 95% confidence intervals for the parameters of the quadratic Taylor's law (4). The substantially and

-	8	0								
	Health							Intercept		
	System	b ₂	lo b ₂	hi b ₂	b1	lo b ₁	hi b1	C ₁	$\log C_1$	hi c1
	1 NYC-a	-1.5587	-1.8352	-1.2823	1.4061	1.3173	1.495	0.5176	0.48978	0.54542
	2 NYC-b	-2.3272	-3.3249	-1.3296	2.8083	2.1197	3.497	-0.03568	-0.13277	0.061417
	3 Chicago-a	-0.71316	-0.9095	-0.51683	1.0527	1.0018	1.1036	0.4169	0.39678	0.43702
	4 Chicago-b	-0.29897	-0.63715	0.039209	1.061	0.97871	1.1433	0.47221	0.45618	0.48824

194 significantly negative estimates of b₂ for New York characterize a concave, non-linear variance function for eCCI.

195

Table 5. Statistics of Taylor's law regressions (3) in Table 3 and quadratic Taylor's law regressions (4) in Table 4: coefficient of determination R²; probability p of the F-statistic of the null hypothesis of no relationship; residual sum of squares RSS. Fcompare is the F-statistic comparing the fit of the quadratic Taylor's law with the fit of Taylor's law; P is the probability of the null hypothesis of no improvement. The null hypothesis is rejected for all four health systems, even though for HS 4 the difference between TL and QTL is visually very small in Figure 3.

201 Taylor's law regressions (3) in Table 3

Health System	\mathbb{R}^2	pFstat2	MSE2	RSS2
1 NYC-a	0.84766	5.09E-27	0.014956	0.9273
2 NYC-b	0.80843	8.02E-27	0.011681	0.81766
3 Chicago-a	0.95394	1.71E-44	0.003822	0.24463
4 Chicago-b	0.94877	1.16E-43	0.002446	0.15898

203	quadratic Taylor's law regressions (4) in Table 4							
	Health System	\mathbb{R}^2	pFstat2	MSE2	RSS2	Fcompare	Р	
	1 NYC-a	0.9506	1.44E-40	0.00493	0.30071	127.1	0	
	2 NYC-b	0.85419	1.41E-29	0.009019	0.62233	21.656	0	
	3 Chicago-a	0.97492	3.80E-51	0.002115	0.13322	52.688	0	
	4 Chicago-b	0.95115	1.11E-42	0.002369	0.1516	3.1191	4.72E-06	

For each HS separately, we tested whether QTL described the data significantly better than TLby an *F*-test:

207 (5)
$$F = \frac{\frac{RSS_{TL} - RSS_{QTL}}{df_{TL} - df_{QTL}}}{\frac{RSS_{QTL}}{df_{QTL}}}$$

where RSS_{TL} , RSS_{QTL} are the residual sums of squares of TL and QTL, respectively, and df_{TL} , df_{QTL} are the corresponding degrees of freedom. Then, if *fcdf* is the cumulative distribution function of the *F* distribution with degrees of freedom df_{TL} , df_{OTL} , we calculate

211 (6)
$$P = 1 - fcdf(F, df_{TL}, df_{QTL}).$$

212 If *P* is larger than a critical value, e.g., 0.05, then QTL is not a significantly better description

of the data than TL. If *P* is smaller than a critical value, then QTL is a significant

214 improvement over TL. The last two columns of Table 5, Fcompare and P, strongly confirm

215 QTL describes the variance functions of all four HSs better than TL, and are significantly

concave for HS 1, 2, and 3.

217 Quantiles of eCCI by age in each health system

The rising trend of the variances of eCCI in Figure 2 indicates increasing scatter ("diversity" in the sense of Hultsch et al. 2002) of eCCI with increasing age. To quantify the increasing scatter further, we display the quantiles of the distribution of eCCI at each age in this section. In the next section, we refine the analysis by fitting the frequency distributions of eCCI at each age to the negative binomial distribution.

223 The quintiles of eCCI for age *a* are those four values of eCCI such that 1/5, 2/5, 3/5, and 4/5 of

the people of age *a* have eCCI less than or equal to those values. These quintiles divide the

225 ranked list of eCCIs into five equal (or nearly equal) portions. The increases with age in the 226 variance of eCCI suggest that the upper quintiles of the distribution of eCCI should rise more 227 rapidly with age than the lower quintiles because the distribution of eCCI should spread out more 228 with increasing age. Figure 4 confirms this suggestion for each HS and for each age with 100 or more people. The four quintiles of eCCI diverge with increasing age. Figure 4 also shows that 229 230 the upper quintiles and the entire distribution of eCCI in middle and upper ages are notably lower 231 in HS 4 than in the other three HSs, for reasons yet to be determined.

- 232
- 233 Figure 4 Four quintiles of eCCI as a function of age, for ages with 100 or more people in that 234 health system. Source: QuantilesCCIbyAgeForEachHS_20250128-173406.png



Quantiles of CCI (vertical axis) as a function of age (horizontal axis)

235

237 Frequency histogram of eCCI within each age group

238 The frequency histogram of eCCI within an age group in a HS shows how many people of that 239 age and HS had eCCI equal to 0, how many had eCCI equal to 1, how many had eCCI equal to 2, 240 and so on. To give statistically credible information about the relative frequency of each value of 241 eCCI by age group, we devised 16 age groups containing four years of age each: 19-22 years 242 (that is, from age 20 minus one up to and including 20 plus two), 23-26 (that is, from age 24 243 minus one up to and including 24 plus two), and so on up to 79-82 (that is, from age 80 minus 244 one up to and including 80 plus two). We label each of these age groups by the age in the group that is divisible by 4, namely, 20, 24, 28, ..., 76, 80. Figure 5 shows the number of people in 245 246 each age group in each HS. As in Figure 1, the larger proportion of people at older ages in HS 2 247 is visible.

249 Figure 5 Number of people in each 4-year age group in each health system. Source:



250 NPeopleByAge16GroupsHealthSystem_20250115-184027.png

253

254 For all 64 of these age groups (16 age groups for each of 4 HSs), the variance of eCCI exceeded 255 the mean eCCI. (When groups of 100 people or more in individual years of age were used to 256 compare the mean and variance of eCCI, a handful of the youngest and extremely oldest single-257 year age groups had a variance of eCCI less than the mean eCCI. Evidently, these few exceptions 258 resulted from sampling fluctuations.)

- 259 The negative binomial distribution is a classical parametric model of over-dispersed integer-
- 260 valued nonnegative random variables. Figures 6-9 show the observed counts of people in HS 1-4,
- 261 respectively, who have each eCCI and the expected counts (green line) from a negative binomial
- 262 distribution fitted to the empirical probability density function (which is just the frequency
- 263 histogram divided by the sum of all counts in the histogram). The agreement is generally

- 264 excellent. Some deviations arise at the largest values of eCCI where fewer people are observed.
- 265 Because the vertical axis (number of people) is on a logarithmic scale, the (rare) zero counts are
- 266 omitted entirely.
- 267

- Figure 6 For HS 1, the number of people in each 4-year age group (vertical axis of black ×) who
- have each value of eCCI (horizontal axis of black \times), and the fitted negative binomial
- 270 distribution (green line). The vertical axis for number of people is on a logarithmic scale. Source:
- 271 PeopleVsCCIbyAgeHS1_20250115-160715.png



Figure 7 For HS 2, the number of people in each 4-year age group (vertical axis of black ×) who
have each value of eCCI (horizontal axis of black ×), and the fitted negative binomial
distribution (green line). The sample variance differs most from the variance predicted by the
negative binomial distribution in the age groups 36-54. The other health systems do not show
similar deviations in these age groups. Source: PeopleVsCCIbyAgeHS2_20250115-160715.png



Figure 8 For HS 3, the number of people in each 4-year age group (vertical axis of black ×) who
have each value of eCCI (horizontal axis of black ×), and the fitted negative binomial
distribution (green line). Source: PeopleVsCCIbyAgeHS3_20250115-160715.png



Figure 9 For HS 4, the number of people in each 4-year age group (vertical axis of black ×) who have each value of eCCI (horizontal axis of black ×), and the fitted negative binomial distribution (green line). Source: PeopleVsCCIbyAgeHS4_20250115-160715.png



287

288

289 Negative binomial distribution

290 The negative binomial distribution can be parameterized in various ways. The probability density

291 function of the negative binomial distribution in Matlab

292 (https://www.mathworks.com/help/stats/nbinpdf.html), which we used for these computations,

is: for 0 , <math>r > 0 (*r* may or may not be an integer),

294 (7)
$$\Pr(eCCI = x) = \left(\frac{\Gamma(r+x)}{\Gamma(r)\Gamma(1+x)}\right)p^r(1-p)^x, x = 0, 1, 2, 3, ...$$

If eCCI is negative binomially distributed as in (7), the expectation or population mean of eCCIis

297 (8)
$$E(eCCI) = \frac{r(1-p)}{p}$$

and the population variance of eCCI is

299 (9)
$$Var(eCCI) = \frac{r(1-p)}{p^2}$$

300 (https://www.mathworks.com/help/stats/nbinstat.html). Two possible variance functions for the

301 negative binomial distribution are

302 (10)
$$Var(eCCI) = \left(\frac{1}{p}\right) E(eCCI)$$
 if *p* is constant and only *r* varies;

303 and

304 (11)
$$Var(eCCI) = E(eCCI) + \left(\frac{1}{r}\right) [E(eCCI)]^2$$
 if r is constant and only p varies.

305 Both (10) and (11) imply
$$Var(eCCI) > E(eCCI)$$
.

306 If (10) holds for every age group *a* with the same value of *p* for all age groups and only *r* varies,

307 then (10) is identical to TL (1) with c = 1/p and b = 1. In the limit as p approaches 1, the

308 variance function (10) approaches Var(eCCI) = E(eCCI), which is the variance function of the

- 309 Poisson distribution, where variance equals mean. If p varies notably with age a, then (1) and
- 310 (10) are not consistent.
- 311 If only *p* varies and *r* is constant, then the variance is a quadratic function (11) of the mean. In
- 312 the limit as r approaches infinity, the variance function (11) again approaches Var(eCCI) =
- 313 E(eCCI), the variance function of the Poisson distribution. In the limit as r approaches 0, the
- second term in (11) dominates the first and (11) is asymptotic to TL (1) with c = 1/r and b = 2.
- 315 For fixed finite *r*, when only *p* varies, *Var(eCCI)* is a convex increasing function (11) of
- 316 E(eCCI), not consistent with TL (1).

Figure 10 plots the estimated negative binomial parameters r, p as a function of age group a for each HS. In all four HSs, with increasing age, r changes more than p. The major trend is that rdecreases from age group 20 to 28 and increases from age group 28 onward (except in HS 4 above age 64). In HS 1 and HS 2 (New York), the estimated r (blue line, blue x) notably exceeds the estimated p (orange line, orange +). As age increases, in HSs 1, 2, and 3, p decreases slightly at the earlier ages and then smoothly levels off, while in HS 4, p falls over the entire range of ages (apart from a slight rise between ages 40 and 48).

324 The trends over age in the estimated parameters of the negative binomial distribution of CCI 325 account qualitatively for the occurrence of Taylor's law with exponent near or less than b = 1 in 326 these data. Using the estimated parameter values r, p, Figure 11 plots the population variance $Var(CCI) = r(1-p)/p^2$ as a function of the predicted mean E(CCI) = r(1-p)/p for each 327 328 age group in each HS (blue x marks) and a (black) curve, a power function corresponding to Taylor's law, fitted by least squares to the 16 values of $(r(1-p)/p, r(1-p)/p^2)$ for the 16 329 330 age groups in each HS. Only for HS 2 is b > 1 significantly. In HS 1 and 3, b < 1 significantly, 331 and b is not significantly different from one in HS 4.

332 For HS 1 (New York) and HS 3 (Chicago) in Figure 11, *Var(CCI)* increases as a concave

function of the mean E(CCI) compared to the fitted power-law curve. This observation is

334 consistent with the curved path of the circles in Figure 3's top left panel for HS 1 (New York).

Likewise, New York's HS 2 gives concordant M-shaped circles for *Var(CCI)* in Figure 3 and

blue x markers in Figure 10, top right panels, compared to the fitted power-law curves. Similarly,

the panels for Chicago in the bottom row of Figure 10, like the panels for Chicago in the bottom

row of Figure 3, roughly support Taylor's power law variance function.



Figure 10 Estimates of the negative binomial parameters r, p as a function of age group a for

each health system. Source: NegativeBinomialParameters_20250115-185504.png

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Figure 11 The variance $Var(eCCI) = r(1-p)/p^2$ of a negative binomial distribution of individual eCCI as a function of the mean E(CCI) = r(1-p)/p for each age group in each health system (blue x marks) and a (black) curve, corresponding to a fitted Taylor's power law (1). Source: NegativeBinomialPredictedTLbyHS_20250117-115016.png



350 **DISCUSSION**

351 Summary and interpretation of main results

We show that, in four health systems (HSs), two in New York and two in Chicago, the variation in the enhanced Charlson Comorbidity Index (eCCI) (defined by Charlson et al., 2025) among people of a given age increases with age, apart from random variation in small samples. The increasing variation is systematically related to an increase with age in the average eCCI, in qualitative agreement with Taylor's power law of fluctuation scaling or the quadratic Taylor's

357	law. The quintiles of eCCI diverge increasingly with age, again revealing increasing scatter in
358	eCCI of individuals of each age. The frequency histogram of eCCI within age groups is generally
359	well described by a negative binomial distribution. The parameters of the negative binomial
360	distribution of different age groups account approximately for the appearance of Taylor's law or
361	the quadratic Taylor's law in these four HSs. The underlying mechanisms of the negative
362	binomial distribution and of Taylor's law or its quadratic generalization remain to be determined.
363	The differences among the four HSs could be due to:
364	• differences in the patient population,
365	• differences in the environment of the different HSs,
366	• differences in the coding practices of different HSs in different cities and states, including
367	differences among clinicians or health insurance companies in their propensity to add
368	comorbid conditions to records of ageing patients who already have multiple comorbid
369	conditions,
370	• differences in the survival of older adults with larger eCCI if the survivors to older ages
371	experienced high values of eCCI less often than those who died,
372	• or to interactions of these and other unnamed factors.
373	In every HS reported here, in every 4-year age group, the variance of eCCI substantially exceeds
374	the mean eCCI. This observation rejects a Poisson model of variation in eCCI, in which the
375	variance of eCCI equals the mean of eCCI. Poisson variation would arise if every cause of an
376	increase by one unit in eCCI were independent of every other cause and had an equal probability
377	of causing an increase in eCCI. The medical conditions that are counted in eCCI are known not
378	to be independent and not to be of identical probability. The sample has more variation in eCCI

than the Poisson model can describe, suggesting that the factors that contribute to eCCI are
positively associated, i.e., they tend to vary together. Moreover, these associations may change
with age.

The clear finding that the variance and the mean of eCCI rise with age means that aging brings increasing comorbidity (known already) and increasing diversity of comorbidity (described here). The upper quantiles of the distribution of eCCI at each age rise more rapidly than the lower quantiles and the mean of eCCI. Increasing age brings more comorbidities than would be predicted from the increase in the mean of eCCI alone. Clinical practice should take account of the increasing diversity of comorbidity.

388 Interpreting the negative binomial distribution

389 The negative binomial distribution describes very well the frequency distribution (histogram) of 390 eCCI within nearly every 4-year age group in all 4 health systems. The negative binomial 391 distribution has at least three well known interpretations or models (Bailey 1964; Grimmett and 392 Stirzaker 2001): heterogeneity, contagion, and sampling.

393 Heterogeneity: the negative binomial is a mixture of Poisson distributions with gamma-

distributed means. In greater detail, if an age group contains subgroups, and each subgroup has a

395 Poisson distribution of eCCI, and the mean eCCIs of the different subgroups vary according to a

396 gamma distribution, then the whole age group has a negative binomial distribution of eCCI.

397 The contagion model's key idea is that the more chronic conditions a person has, at any age, the

398 higher the risk that the person will acquire an additional chronic condition. This hypothetical

399 "contagion" (in a statistical sense, not related specifically to infectious diseases) among

400 conditions could be tested empirically with adequate longitudinal data.

Sampling: This interpretation is possible when the parameter r is a positive whole number. 402 Suppose that, over time, each person experiences a succession of independent random trials. 403 Suppose a person's trial "succeeds" with probability p and "fails" with probability 1 - p. To 404 achieve r successes, a person has to experience a variable number x of failures since there is a 405 fixed positive probability 1 - p of failure at every trial. The number of extra trials x a person 406 experiences to reach r successes is a random quantity that obeys the negative binomial 407 distribution (7). Here the number of failures x corresponds to eCCI, the number of comorbid conditions a person acquires until achieving r "successes." 408

409 The available data are insufficient to select among these possible mechanisms that lead to the

410 negative binomial distribution. The multiple possibilities suggest questions for future research.

411 *Future research*

401

412 For clinical applications, to identify risk groups and opportunities for patient education and 413 intervention, it would be highly desirable to repeat this analysis after stratifying each age group 414 by sex, marital status, education, behavior (smoking, drug use, alcohol use, exercise, sleep, diet, 415 relationships), socio-economic status, environmental hazards, and perhaps other characteristics. 416 The auxiliary variables associated with individuals of a given age with high and low eCCI may 417 offer clues to possible preventive interventions or behavioral changes.

The 39 conditions included in the enhanced eCCI generate $2^{39} \approx 5.5 \times 10^{11}$ (almost 550 billion) 418

419 possible combinations of conditions. Identifying each patient's combination of conditions would

420 make it possible to examine the marginal frequency of each condition and the associations,

421 positive and negative, among conditions.

422 The present analysis is cross-sectional, comparing individuals who are observed once only at 423 different ages. Longitudinal records of medical history for the oldest age groups would make it 424 possible to calculate the eCCI retrospectively for those individuals, say, at each age 10, 20, 30, 425 ... years earlier, and to compare the trajectories (frequency histogram, mean, variance, and 426 relation of variance to mean) over increasing age of individuals at an advanced age who have 427 low eCCI with the trajectories of individuals who have high eCCI. Longitudinal records of 428 medical history would also make it possible to evaluate the influence of selective or differential 429 survival on the distribution of eCCI among the living, and easier to discriminate among 430 alternative mechanisms that lead to the observed negative binomial distribution of eCCI within 431 age groups.

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