How do Mortality Differences Across the U.S. Matter?

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7 Abstract

8 BACKGROUND

⁹ There are large mortality gaps among U.S. states measured by life ex-¹⁰ pectancy, lifespan variance, and crude death rate. Yet these measures tell ¹¹ different stories, since they are affected differently by mortality change ¹² across space and time.

OBJECTIVE

Identify the contributions of age-specific mortality rate differences to gaps
 in the three aggregate measures.

16 METHODS

We make novel use of decompositions to analyze how age-specific mortality differences determine each of the three mortality measures. For life expectancy and lifespan variance, we separate the size of age-specific mortality differences from the sensitivity to those differences. For crude death rate, we separate exactly the age-specific contributions of population age structure, and mortality rates.

23 CONCLUSIONS

- ²⁴ Differences in under-5 mortality do not significantly affect differences in
- ²⁵ any measure. Mortality differences at middle and old ages play a signifi-
- ²⁶ cant role in differences in life expectancy. In contrast, differences in lifes-

pan variance are more affected by mortality differences at the middle ages
than at old ages. The significance of mortality differences differs in the
two sexes. Crude death rates are largely driven by the elderly, with crossstate differences primarily explained by the share of the older population,
while the role of mortality differences has grown over time.

32 CONTRIBUTION

We use a sensitivity-based decomposition approach and the Kitagawa decomposition to analyze gaps in three aggregate mortality indicators. Our approach is essential because an age group's contribution to a gap in a given aggregate indicator can result from large mortality rate differences, high sensitivity of the indicator, or shift in population composition.

38 1. Introduction

Mortality differences across U.S. states have long been recognized (Sub-39 ramanian, Kawachi, and Kennedy 2001; Fenelon 2013; Montez and Berk-40 man 2014). To assess the import of these differences, three distinct met-41 rics have been widely used: life expectancy, the variance of age-at-death, 42 and the crude death rate. Life expectancy, the average number of years 43 a person is expected to live, often serves as a key indicator of population 44 health, used to track improvements in health outcomes and guide policy 45 (Avendano and Kawachi 2014; Oeppen and Vaupel 2002). The variance 46 of age-at-death reveals the degree of inequality within a population, offer-47 ing insights into the equity of survival outcomes, and has attracted growing 48 attention (Wilmoth and Horiuchi 1999a; Edwards and Tuljapurkar 2005; 49 Firebaugh et al. 2014; Van Raalte, Sasson, and Martikainen 2018). Finally, 50 the crude death rate provides a direct measure, the number of deaths per 51 1,000 people in a population over a given time period, and is often featured 52 in media reports and public discourse (Xiao, Mei, and Jiang 2022; Liang, 53 Guo, and Tuljapurkar 2023). Even though crude death rate is known to de-54 pend greatly on population age distribution, this metric remains useful for 55 monitoring changes in mortality over time and across geographic units. Of 56 these three, no single metric captures the distinct, albeit complementary, 57 dimensions of population mortality (Aburto et al. 2020). 58

A particular state in the U.S. may have higher life expectancy than 59 the U.S., but also a higher variance of age-at-death and a higher crude 60 death rate. How should we evaluate these metrics in terms of the changes 61 within a state over time, or in comparison to other states? Any comparative 62 analysis must reckon with the fact that declines in age-specific mortality 63 may have quite different effects on these three metrics. Reductions in age-64 specific mortality at any age always increase e_0 (Keyfitz, Caswell et al. 65 2005; Goldman and Lord 1986; Vaupel and Canudas-Romo 2003). But 66 the sensitivity of variance of age-at-death to mortality decline depends on 67 a "young-old threshold" age. Mortality declines at ages below the thresh-68 old will reduce the variance but mortality decline at ages above the thresh-69

old will increase the variance! The reason is simply that saving lives at 70 young ages narrows the distribution of ages at death, while saving lives 71 at old ages pushes the frontier of longevity outward, broadening the dis-72 tribution (Edwards and Tuljapurkar 2005; Vaupel, Zhang, and van Raalte 73 2011; Zhang and Vaupel 2009; Van Raalte and Caswell 2013; Gillespie, 74 Trotter, and Tuljapurkar 2014). Empirical studies confirm these patterns: 75 historically, mortality declines at younger ages consistently raised e_0 while 76 reducing the variance of age-at-death (Wilmoth and Horiuchi 1999a; Ed-77 wards and Tuljapurkar 2005). More recent gains at older ages above the 78 threshold have been linked to widening lifespan inequality or slower com-79 pression (Tuljapurkar, Li, and Boe 2000; Van Raalte, Sasson, and Mar-80 tikainen 2018). The crude death rate is influenced not only by mortality 81 decline but also by changes in population age structure (Preston, Heuve-82 line, and Guillot 2001). Even with reductions in age-specific mortality 83 rates, an aging population can lead to higher crude death rates. 84

Numerous methods have been developed to quantify age-specific con-85 tributions to changes in life expectancy (Gupta 1978; Pollard 1982; Ar-86 riaga 1984; Andreev, Shkolnikov, and Begun 2002; Shkolnikov, Andreev, 87 and Begun 2003) and some have been developed to explore changes in 88 lifespan variance (Andreev and Shkolnikov 2012; Gillespie, Trotter, and 89 Tuljapurkar 2014). When comparing national populations, a general find-90 ing is that e_0 is most affected by mortality among children under five and 91 adults over 60, whereas variance of age-at-death is most affected by mor-92 tality at middle ages, such as around age 40 (Hiam, Minton, and McKee 93 2021; Aburto et al. 2020). But do these conclusions also hold for state 94 within the U.S.? 95

To answer that question, we decompose spatial gaps in life expectancy at birth and variance of age-at-death in terms of: (a) the differences in age-specific mortality rate and (b) the sensitivity to mortality (change of measures to proportional changes in age-specific mortality rate). In this way, we can identify the ages at which mortality differences truly matter by evaluating mortality gaps and sensitivity together. An age group will only matter if it has both a large sensitivity and a large differences in mortality rate. If either one of these is close to zero, that age group does
not contribute significantly. In contrast to many previous decompositions,
our approach makes clear why mortality decline in a given age group can
have a high contribution to one mortality metric but a much smaller (even
negative) contribution to another metric.

Secondly, in the same spirit, we examine differences in crude death rate using the Kitagawa decomposition to separate exactly the contributions of differences in age-specific mortality rates and in population age structure between populations. Unlike life expectancy at birth and variance of age-at-death, the crude death rate is sensitive to differences in the age structure of populations, so we are careful in making comparisons across space and time.

The need for new decompositions was noted recently by Su and col-115 leagues (2024), who developed a different decomposition method for life 116 expectancy and life disparity and applied that to U.S. states. However, in 117 their approach the population composition is determined by the proportion 118 of each state's population in a given age group relative to the national pop-119 ulation. Consequently, their decomposition reflects the relative weight of 120 each state's mortality in shaping national-level outcomes, and so is highly 121 sensitive to mortality in populous states. As a result, even if small states 122 achieve notable mortality improvements, their impact on national life ex-123 pectancy growth may appear limited. Our approach does not have that 124 problem. To highlight state-level mortality dynamics, we use decomposi-125 tions that avoid the influence of population size and are both mathemati-126 cally and demographically meaningful. 127

128 **2. Data source**

We obtain age- and sex-specific mortality rates for 50 U.S. states (exclude the District of Columbia) from 1969 to 2019 in each single year through the United States Mortality DataBase (USMDB 2024). State-level population data for the same period were sourced from the National Cancer Institute (NCI 2024). To match population data and mortality data across
all ages, the penalized composite link model (PCLM) from R package *'ungroup'* is used to extend and smooth the population counts to age 110+
(Rizzi, Gampe, and Eilers 2015).

137 3. Methods

3.1 Sensitivity of life expectancy at birth and variance of age-at-death

139 3.1.1 Life expectancy at birth

In a given population, the life expectancy at birth e_0 is

$$e_0 = \int_0^\infty dx \, l(x),\tag{1}$$

where l(x) is the probability of surviving from birth to age x.

In any year, for any state *i*, the age specific mortality rate $\mu^i(x)$ at age *x* can always be thought of relative to the national mortality rate $\mu^u(x)$,

$$\mu^{i}(x) = [1 + \delta^{i}(x)]\mu^{u}(x).$$
(2)

This equation defines the proportional change $\delta^i(x)$ that must be applied to the national $\mu^u(x)$ to produce $\mu^i(x)$.

If the product $(\delta^i(x) \cdot \mu^u(x)) \ll 1$, we can use a linear Taylor expansion to get the life expectancy for state *i*,

$$e_0^i = e_0^u - \int_0^\infty dx \, l^u(x) \, \int_0^x ds \, \delta^i(s) \mu^u(s), \tag{3}$$

where $e^{u}(0)$ is the life expectancy for the nation. Thus the contribution of mortality rate differences at any age x is the product

$$-l^u(x) \int_0^x ds \,\delta^i(s)\mu^u(s),\tag{4}$$

Is the linear Taylor expansion appropriate? We examine the distribution of the product $(\delta^i(x) \cdot \mu^u(x)) \ll 1$ for all states in the U.S., for all years, for both sexes, and found that the product is almost always much less than 1 (see the details in Appendix Table A-1). Not surprisingly, we find (below) that the decomposition equation (3) is remarkably accurate.

Using California females in 1969 and 2019 as examples, the observed 155 proportional difference in mortality rate between that state and the U.S. 156 is shown in Figure 1a. In 2019, female mortality rates in California were 157 consistently lower than in the U.S. Figure 1b displays age-specific contri-158 butions to the life expectancy gap, computed using equation (3). Finally, in 159 Figure 1c we sum the age-specific contributions to obtain the total change 160 in e_0 relative to the baseline U.S. females. The estimated California fe-161 male e_0 values is very close to the observed values, demonstrating that our 162 method works for the observed data. 163





Note: The figures show the decomposition steps for the e_0 gaps, with California and U.S. females in 1969 and 2019 as examples. (a) The proportional differences between $\mu^{CA}(x)$ and $\mu^u(x)$, denoted $\delta^{CA}(x)$, are calculated according to equation (2). (b) The age-specific contributions to the e_0 gaps between California and the U.S.. (c) A comparison of the observed e_0^{CA} and the estimated e_0^{CA} , where the estimated e_0^{CA} equals e_0^u plus the changes in e_0^u due to proportional changes $\delta^{CA}(x)$ in $\mu^u(x)$. *Source*: Authors' calculation based on USMDB (2024).

164 **3.1.2 Variance of age-at-death**

We adopt a similar strategy to decompose the variance of age-at-death. Focus on the variance of age-at-death for deaths after age a denoted as V(a),

$$V(a) = \frac{2}{l(a)} \int_{a}^{\infty} dx \, x l(x) - [e(a)]^{2} - 2ae(a)$$
(5)

where l(a) is the conditional survival probability from age a to age x, and e(a) is the life expectancy at age a.

For any state *i*, we can compute $\delta^i(x)$ as the age-specific proportional change in the national mortality rate, as defined in equation (2). Then a linear Taylor expansion of equation (5) yields the difference between the state's variance $V^i(a)$ and the national variance $V^i(a)$ as

$$V^{i}(a) = V^{u}(a) + 2\int_{a}^{\infty} dx \left[e^{u}(x) + a - x\right] \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \,\delta^{i}(s)\mu^{u}(s).$$
 (6)

Here the contribution of mortality rate differences at any age x is the product

$$2[e^{u}(x) + a - x]\frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \,\delta^{i}(s)\mu^{u}(s).$$
(7)

We focus on lifespan variance for deaths past early childhood, with a = 6. We do so to exclude the disproportionate (but uninformative) influence of early childhood mortality on variance of age-at-death (Edwards and Tuljapurkar 2005). We could alternately have used V(10) (Edwards and Tuljapurkar 2005; Engelman, Canudas-Romo, and Agree 2010) or V(15) (Vaupel, Zhang, and van Raalte 2011; Gillespie, Trotter, and Tuljapurkar 2014). If she prefers, the reader can use any other choice.

To illustrate, the age-specific contributions to the V(6) gaps between California and the U.S., as well as the estimated V(6) values, are shown in Figure 2b and Figure 2c respectively for illustration.

Figure 2: Illustration of the V(6) decomposition method



Note: The figures show the decomposition steps for the V(6) gaps, with California and U.S. females in 1969 and 2019 as examples. (a) The proportional differences between $\mu^{CA}(x)$ and $\mu^u(x)$, denoted $\delta^{CA}(x)$, are calculated according to equation (2). (b) The age-specific contributions to the V(6) gap between California and the U.S. average. (c) A comparison of the observed $V^{CA}(6)$ and the estimated $V^{CA}(6)$, where the estimated $V^{CA}(6)$ equals $V^u(6)$ plus the changes in $V^u(6)$ due to proportional changes in $\mu^u(x)$. *Source*: Authors' calculation based on USMDB (2024).

3.1.3 Summary of the sensitivity approach

In our decompositions above, the U.S. national population is always the reference. We identify age-specific contributions to the differences in these measures in terms of the product of the sensitivity of the measure to the age-specific mortality rate $\rho(\mu(x))$, and the proportional difference in agespecific mortality rates $\delta(\mu(x))$ between the populations being compared. For an age group to matter, we need a large sensitivity with a large mortality gap.

¹⁹⁴ **3.2** Crude death rate and Kitagawa decomposition

The crude death rate (*CDR*), also known as the per-capita death rate, is defined as:

$$CDR = \int_0^\infty dx \,\mu(x) \cdot c(x),\tag{8}$$

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where $\mu(x)$ is the mortality rate and c(x) the proportion of individuals at age x. This expression shows how the CDR is determined by the interaction of age-specific mortality rates and the population's age structure.

We use the Kitagawa decomposition (Kitagawa 1955) to separate agespecific mortality differences and compositional differences in explaining the *CDR* gap between an individual state (indicated by superscript i) and the U.S. (indicated by superscript u) as a whole:

$$\Delta CDR = \int_0^\infty dx \, [\mu^i(x) - \mu^u(x)] \frac{c^i(x) + c^u(x)}{2} + \int_0^\infty dx \, \frac{\mu^i(x) + \mu^u(x)}{2} [c^i(x) - c^u(x)]$$
(9)

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The first term on the right-hand side of the above equation isolates the 206 contribution of differences in age-specific mortality rates between the indi-207 vidual state i and the U.S. average u, by holding the age structure constant 208 at the average over both populations. The second term isolates the contri-209 bution of differences in age structure between the state *i* and the U.S., by 210 holding mortality rates constant at their averaged across both populations. 211 This term highlights how differences in population structure, such as in the 212 proportion of elderly individuals, affects the CDR gap. 213

For instance, states with higher proportions of older populations, $c^i(x) > c^u(x)$ at older ages, may exhibit higher CDRs, even if their age-specific mortality rates $\mu^i(x)$ are similar to the U.S. average. Conversely, states with much lower age-specific mortality rates, $\mu^i(x) < \mu^u(x)$, may offset the impact of an older population structure, resulting in a lower CDR.

219 4. Results

220 4.1 Trends in mortality dynamics across U.S. states

We start with the important aspects of the similarities and differences 221 across the country and also over time. While life expectancy has increased 222 in all states, though at different speeds, changes in the variance of age-at-223 death and crude death rate have varied in both direction and magnitude, 224 with some states experiencing increases and others declines. These pat-225 terns reveal that some states have maintained and expanded their advantage 226 in all three aspects over time, in contrast to some that are always disadvan-227 taged. But most states have followed complex trajectories over time, with 228 improvements in one indicator but stagnation or deterioration in others. 229

All three maps in Figure 3 use green to indicate an advantage (higher 230 life expectancy, lower lifespan variance, and lower crude death rate) of a 231 individual state compared to the U.S. average, and use purple to indicate 232 the opposite. As a reference, in 2019, U.S. females had a e_0 of 81.48 years, 233 a V(6) of 203.71, and a CDR of 8.68 per 1,000 people. For U.S. males, e_0 234 was 76.46 years, V(6) was 261.70, and the CDR was about 9.17 per 1,000 235 people. In general, states on the West Coast and in the Northeast exhibited 236 relatively higher life expectancy, lower variance of age-at-death, and lower 237 crude death rates compared to the U.S. average, while Southern states and 238 parts of the Central region showed the opposite pattern. 239

Figure 3: The gaps between individual states and the U.S. in 2019



Note: For gaps in e_0 and *CDR*, we directly use the differences between individual states and the U.S.. In contrast, the percentage differences in V(6) are calculated as $[V^i(6) - V^u(6)]/V^u(6)$, given that the absolute values of variance are much larger than those of life expectancy or the crude death rate.

Source: Authors' calculation based on USMDB (2024).

Figure 4 illustrates state-level mortality dynamics from 1969 to 2019. Each point represents a specific state or the U.S. average (latter identified

on the figure). The horizontal movement shows that e_0 improved in all 242 states from 1969 to 2019. The vertical axis represents changes in the CDR, 243 with points distributed both above and below the horizontal reference line. 244 reflecting that some states experienced an increase in CDR over the past 50 245 years, while others saw a decline. The size and color of each point reflect 246 changes in V(6): green indicates a decrease in V(6), signifying a narrower 247 distribution of age at death and reduced lifespan inequality, whereas purple 248 indicates an increase in V(6), signifying increased lifespan inequality. The 249 magnitude of these changes is shown by the varying size of the points. 250 While V(6) declined for females in most (but not all) states, it increased 251 for males in most (but not all) states. 252

Figure 4: Mortality dynamics of each state and the U.S., from 1969 to 2019



Note: The changes of mortality indicators here are calculated by values in 2019 minus values in 1969. Details of each state show in Figure A-3. *Source*: Authors' calculation based on USMDB (2024).

We illustrate the diversity of state-level mortality change by three examples. In 2019, California had higher e_0 , lower V(6), and a lower CDR

than the U.S. average. Over the 50-year period, California also experi-255 enced improvements in all three measures, indicating steady progress in 256 mortality outcomes. West Virginia, by contrast, exhibited significantly 257 lower life expectancy at birth, higher variance of age-at-death, and higher 258 crude death rate relative to the national average. From 1969 to 2019, West 259 Virginia's gains in e_0 were among the smallest across all states, and were 260 accompanied by rising CDR and a marked increase in V(6). Texas pre-261 sented a picture of overall improvement between 1969 and 2019, with in-262 creasing e_0 , declining V(6) and declining CDR. In 2019, all three mea-263 sures in Texas were lower than the national average. These mixed patterns 264 reinforce the need for multiple indicators to fully assess mortality condi-265 tions. 266

4.2 Age-specific contribution to mortality gaps

We now apply our decompositions to examine how mortality differences 268 by age shape state-level performance as measured by the three metrics e_0 , 269 V(6), and CDR in the years 1969 to 2019. The following generalizations 270 emerged from our analyses. (1) Under-five mortality contributes negligi-271 bly to state-level differences in e_0 and crude death rates, as improvements 272 in child mortality have benefited all states relatively equally across the 273 country. (2) Mortality differences at middle and older ages are the primary 274 drivers of differences in e_0 , with midlife playing a more pronounced role 275 for males than for females. (3) V(6) is more strongly influenced by mor-276 tality differences at midlife than at older ages. State-level differences in 277 variance are more pronounced for males than for females. (4) Crude death 278 rates are largely driven by the elderly population in nearly all U.S. states. 279 Among the elderly, cross-state differences in crude death rates are primar-280 ily explained by differences in the degree of population aging, although 281 the influence of mortality differences has increased over time. 282

283 4.2.1 Life expectancy at birth

We start with e_0 , reporting the significance of mortality differences in 284 broad age groups rather than by exact ages. In most states we find minimal 285 contributions from the youngest age group (0-5 years) for both sexes (for 286 more details on this, see Figure A-5, so we focus on middle-ages (6-64 287 years) and old-age (65+ years), where mortality plays a central role in 288 shaping differences in e_0 . From 1969 to 2019, the contribution of old-age 289 mortality increased across most states for both sexes, while midlife mor-290 tality has a stronger impact among males than females. 291

The maps in Figure 5 show contributions in 1969 and 2019 (mid-292 dle age, top row; old age, bottom row) for these two age groups for fe-293 males. Increasingly dark shades of red indicate increasing magnitudes of 294 age-group contributions to the statewise e_0 gaps. Small contributions are 295 those that account for less than 25% of the total e_0 gap, moderate contri-296 butions range from 25% to 75%, and strong contributions exceed 75%. 297 In 1969, contributions from old-age mortality differences were already 298 slightly greater than those from middle-aged mortality among females in 299 most states. By 2019, this pattern had become more pronounced, with old-300 age mortality differences playing an even larger role in many states. For 301 males, contributions from middle-aged and old-age mortality differences 302 were roughly similar in 1969. By 2019, this pattern remained largely con-303 sistent, although contributions from old-age mortality had become slightly 304 more prominent. (Details are provided in Appendix Figure A-5.) 305

Figure 5: Contributions (magnitude only) to gaps in female e_0 of mortality differences at middle and old age groups; individual states vs. the U.S.,1969 and 2019



Note: Different shades indicate the magnitude of contributions to e_0 gaps: slight (<25%), moderate (25%–75%), and strong (>75%). *Source*: Authors' calculation based on USMDB (2024).

306 4.2.2 Variance of age-at-death

Given our finding that the youngest ages contribute the least to mortality differences across states, we focus on the variance of age-at-death for individuals surviving beyond age 6. The sensitivity of V(6) is negative below and positive above a threshold age (T(6)), as illustrated in Figure A-1b. We find that the gaps in V(6) were largely shaped by mortality before the threshold age, whereas the impact of mortality after the threshold age was comparatively smaller and more evenly distributed across states.

Using maps, Figure 6 shows for females the age-specific contribution to percentage differences in inequality V(6) (state vs. nation). Using the year-specific threshold age for U.S. females, we show contributions from deaths before ages below T(6) (top row) and later ages (bottom row). The legend shows how the contributions are grouped, negative percentage differences are marked in green, while positive differences are shown in purple. Negative gaps, i.e., increased inequalities (as measured by V(6)), are mainly driven by deaths occurring before the young-old threshold age, as evidenced by the patterns in the top row of Figure 6.

In 2019, the purple shading in southern states highlights the substantial 323 impact of deaths occurring before the threshold ages on lifespan variance. 324 In contrast, mortality differences after the threshold age (bottom row of 325 Figure 6) are smaller and more evenly distributed across states. For ex-326 ample, in 2019, the V(6) for females in West Virginia was 265.85, about 327 30.51% higher than the national average, indicating a more unequal distri-328 bution of age at death in West Virginia than in the U.S. for females. This 329 difference in equality was the result of a 41.72% difference in early deaths 330 and a -11.21% difference in later deaths. In words, compared to the na-331 tional average, females in West Virginia had a higher probability of death 332 at young ages below T(6), and lived shorter (higher average mortality) 333 than the U.S. at ages above T(6). 334

Figure 6: The decomposition of percentage differences of V(6)between individual states and the U.S. for females



Note: The percentage differences in V(6) between individual states and the U.S. are calculated as $[V^i(6) - V^u(6)]/V^u(6)$, and the percentage differences from inequality before and after threshold age are also divided by $V^u(6)$. *Source*: Authors' calculation based on USMDB (2024).

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4.2.3 Crude death rate

Given the sensitivity of the crude death rate to the age structure of the 336 population, and within the context of population aging, we focused on the 337 crude death rate at ages 65 and above (CDR_{65+}) and explored its contribut-338 ing factors. Our findings highlight a fundamental demographic pattern: by 339 2019, aging populations had become widespread in all states, resulting in 340 deaths increasingly concentrated among older age groups. Cross-state dif-341 ferences in crude death rates among the elderly primarily reflect varying 342 degrees of population aging, though the influence of mortality differences 343 has increased over time. Southern states, in particular, have seen higher 344 crude death rates, might due to the domestic migration of older adults 345 seeking warmer climates. 346

Figure 7 illustrates the strong association between the percentage of 347 the population aged 65 and above and the percentage of CDR_{65+} relative 348 to the total crude death rate CDR_{total} , presented separately for females 349 (left panel) and males (right panel). Each state is represented by individ-350 ual points, with red circles denoting data from 1969 and blue triangles 351 from 2019. The national averages for each year are shown by black mark-352 ers. Several key trends emerge from the figure. First, there is a clear 353 positive association, indicating that states with higher degree of popula-354 tion aging consistently have a larger percentage of CDR_{65+} . Second, in 355 1969, there was considerable variation among states in both the propor-356 tion of elderly populations and the percentage of deaths occurring at ages 357 65 and above. By 2019, this variation had notably narrowed, reflecting 358 widespread population aging across all states and a uniformly higher con-359 centration of deaths among older individuals. Third, across both years, the 360 percentage of deaths among older adults remained consistently higher for 361 females than for males, highlighting women's longevity advantage and the 362 greater concentration of their deaths at older ages. 363

Figure 7: Population aging and the percentage of CDR_{65+} over CDR_{total} by states in 1969 and 2019



Note: The percentage of x-axis equals to $[(N_{65+}/N_{total}) \cdot 100\%]$, where N denotes the population size. The percentage of y-axis equals to $[(CDR_{65+}/CDR_{total}) \cdot 100\%]$. Individual states appear as red circles (1969) or blue triangles (2019), with national averages in black. Source: Authors' calculation based on USMDB (2024).

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Using the Kitagawa decomposition, an age group can substantially affect a CDR difference if it shows large differences between the state and 365 the U.S. in either its population share or its mortality rate. However, mor-366 tality at the old-ages is higher than at middle-ages according to the distri-367 bution of age-st-death, so the CDR is always going to be strongly affected 368 by population composition gaps at old age. And in an aging population, 369 the effect of mortality gaps should be larger when the gaps occur at older 370 ages than middle age. 371

Therefore, we separate CDR_{65+} gaps between states and the U.S. into 372 contributions due to differences in age-specific mortality rates (top row), 373 and those due to differences in population aging (bottom row) in 2019 by 374 gender, shown in Figure 8. Here, purple indicates a positive contribution, 375

and green indicates a negative contribution, with darker shading represent-376 ing larger magnitudes. For illustration, in 2019, the CDR_{65+} for females 377 was 5.28 per thousand in Texas compared to 7.02 per thousand nationally, 378 resulting in a gap of -1.74. This gap can be decomposed into a differ-379 ence due to age-specific mortality rates (0.30, contributing -17%) and a 380 difference due to the population share aged 65 and older (-2.04, contribut-381 ing -117%). The numbers indicate that while Texas has slightly higher 382 mortality rates for elderly females than the national average, its smaller 383 proportion of elderly residents (younger population age structure) drives 384 the lower CDR_{65+} value. 385

As we might expect, in most states, differences in the share of the elderly population account for a larger portion of the differences in CDR_{65+} than differences in old-age mortality levels themselves, while the role of mortality differences has also become increasingly important over time (as confirmed by Figures A-8).

Notably, the contribution from population aging is particularly pronounced in southern states. This might reflect the domestic migration of elderly populations who tend to move from the colder areas in the Northeast and the Midwest to warmer southern states. For instance, Florida has gained the largest numbers of domestic migrants aged 65 years and older between 2015 and 2019 (Mateyka and He 2022).

An interesting gender-specific phenomenon emerges in states such as 397 Arizona: for females, lower CDR_{65+} compared to the national average 398 is mainly driven by lower mortality rates despite slightly higher elderly 399 population shares, while for males, higher CDR_{65+} reflects larger elderly 400 population shares despite lower mortality rates. Taking Arizona in 2019 401 as an example, females show a negative contribution from differences in 402 age structure but a positive contribution from differences in mortality dif-403 ferences. But males display the opposite: a positive contribution from 404 differences in age structure and a negative contribution from differences in 405 mortality. These patterns become clear when examining the specific num-406 bers. For females, the CDR_{65+} was 6.64 per thousand compared to 7.02 407 nationally, yielding a gap of -0.38. This gap decomposes into a difference 408

due to age-specific mortality rates (-0.45, contributing 118%) and popu-409 lation share aged 65+(0.07, contributing -18%). Thus, the lower female 410 CDR_{65+} in Arizona compared to U.S. was primarily driven by lower mor-411 tality rates despite slightly higher elderly population shares. Conversely, 412 for males, Arizona's CDR_{65+} was 6.61 per thousand versus 6.31 nation-413 ally, resulting in a gap of 0.30. This gap decomposes into mortality rate 414 differences (-0.66, contributing 322%) and population share differences 415 (0.96, contributing -222%). Therefore, Arizona's higher male CDR_{65+} 416 compared to U.S. was attributed to substantially higher elderly population 417 proportions, despite lower mortality rates. These findings align with e_0 418 decomposition results, as e_0 in Arizona exceeded the national average for 419 both sexes in 2019, with positive contributions from the 65+ age group. 420

Figure 8:The decomposition of CDR_{65+} gaps between
individual states and the U.S., 2019



Source: Authors' calculation based on USMDB (2024).

421 4.2.4 How mortality differences by age groups drive state-level 422 mortality gaps

Here we summarize how age-specific mortality differences, and their changes
from 1969 to 2019, have shaped state-level differences and trends in the

three metrics: e_0 , V(6), and CDR. Child mortality has played a diminishing role in recent decades, while differences in mortality at middle and older ages, along with variation in population age structures, have become the primary drivers of differences between states (The pattern of change in the metrics is detailed in Figures A-5, A-7, and A-8).

Leading states maintained and widened their edge in mortality metrics. California, for example, is "better" than the U.S. in e_0 , V(6), and CDR in 1969 and 2019. Higher life expectancy primarily results from significant mortality reductions at midlife and older ages; lower lifespan variance reflects persistently low mortality at middle ages; and low CDRcomes from lower age-specific mortality rates combined with a younger population structure.

In contrast, initially disadvantaged states continued to lag, facing mor-437 tality burdens at middle and older ages. A typical example is West Vir-438 ginia, which experienced minimal gains in life expectancy at birth, ris-439 ing crude death rates, and increased lifespan variance. Midlife mortal-440 ity significantly contributed to the e_0 gap between West Virginia and the 441 U.S. in 1969, shifting to poorer old age mortality in 2019, possibly due to 112 variation in health care. A consistently higher lifespan variance resulted 443 mainly from the higher mortality rates among middle-aged groups, largely 444 attributable to causes such as opioid overdoses (Merino et al. 2019). The 445 higher crude death rates in West Virginia in both 1969 and 2019 resulted 446 from a combination of higher mortality rates and a larger proportion of 447 older individuals. A similar and increasing mortality disadvantage is found 448 in parts of Appalachia and the South, where midlife health outcomes have 449 stagnated or worsened. 450

There are also mixed situations. For instance, Mississippi had lower e_0 and higher V(6) compared to the U.S. average, although its own lifespan variance slightly decreased over time. Nevertheless, we find that reducing midlife mortality in Mississippi should remain a key policy target for narrowing lifespan inequality. Note also that females in some states, such as Vermont, performed "better" than the U.S. average in terms of a decreasing V(6) and an increasing e_0 , yet experienced higher and rising crude death rates due to faster population aging. The pattern exhibited by a given state
depends on the metric used, highlighting the importance of our decompositions in uncovering the significance of age-specific mortality change.

461 **5.** Conclusion

Our study used three complementary mortality indicators, life expectancy at birth, lifespan variance, and crude death rate, to reveal distinct consequences of state-level mortality gaps (compared with the United States) from 1969 to 2019. We used innovative decomposition methods to determine how age-specific mortality gaps influenced the three indicators at the state-level.

We examined differences over time between states and the U.S. in 468 e_0 , V(6), and CDR. Some states had relatively high life expectancy but 469 elevated crude death rates due to older population structures. Other states 470 had life expectancies comparable to the U.S. but higher lifespan variability, 171 mainly due to premature deaths, although there is also an effect of longer 472 lives in some states. Despite overall progress in mortality, southeastern 473 states, including Mississippi and West Virginia, continued to exhibit sig-474 nificant inequalities in life expectancy, lifespan variance and CDR gaps. 475

Our decompositions demonstrated that young ages (0-5 years) consis-476 tently contributed minimally to state-level differences in any metric. How-477 ever the contributions of mortality gaps at middle (6-64) and older (65+) 478 ages shaped these differences, sometimes increasingly over time. South-479 eastern states, such as Mississippi and West Virginia, persistently showed 480 the dual burden of rising midlife and old-age mortality. Population age 481 structure differences explained a substantial portion of state-level differ-482 ences in crude death rates, particularly driven by older age groups. 483

Our analysis and decomposition methods disentangle the effects of age-specific mortality differences, the sensitivity of mortality indicators to proportional changes in mortality rates, and population age structure. Agespecific differences in mortality rates may be easy to measure but their

consequences require the analyses we do here. The contribution of any 488 age group to an overall mortality indicator can stem from large differences 489 in mortality rates, high sensitivity of the indicator to even small changes, 490 or shifts in population age structure. By identifying these key drivers of 491 mortality differences, the study help ensure that policy responses can be 492 targeted appropriately – to not only increase how long people live, but also 493 reduce inequalities in lifespan and manage the burden of mortality in each 494 state. 495

496 **References**

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681 Appendices

Detailed derivations of variance of age-at-death

Focus on the variance of age-at-death after age a denoted as V(a), so for a random age at death T we have

$$V(a) = \operatorname{Var}[T] = \operatorname{E}[T^2] - (\operatorname{E}[T])^2,$$
 (10)

where E denotes the expected value and $T \ge a$. The moments are

$$E[T^{2}] = \frac{2}{l(a)} \int_{a}^{\infty} dx \, x l(x) \, + \, a^{2}, \tag{11}$$

$$E[T] = a + e(a) = a + \frac{1}{l(a)} \int_{a}^{\infty} dx \, l(x),$$
(12)

686 and so

$$V(a) = \frac{2}{l(a)} \int_{a}^{\infty} dx \, x l(x) - [e(a)]^{2} - 2ae(a).$$
(13)

For state *i* define the proportional change from U.S. mortality as $\delta^i(x)$, as we did in equation (2). Then for state *i*, the cumulative mortality $M^i(x)$ and the survival probability $l^i(x)$ from birth to age *x* of are respectively

$$M^{i}(x) = \int_{0}^{x} ds \,\mu^{i}(s) = M^{u}(x) + \int_{0}^{x} ds \,\delta^{i}(s)\mu^{u}(s), \qquad (14)$$

$$l^{i}(x) = \exp[-M^{i}(x)] = \exp[-M^{u}(x) - \int_{0}^{x} ds \,\delta^{i}(s)\mu^{u}(s)].$$
(15)

690

For state i, the conditional survival probability from age a to age x is

691 (to linear order in $\delta^i(x)$),

$$\frac{l^{i}(x)}{l^{i}(a)} \simeq \frac{l^{u}(x)}{l^{u}(a)} [1 - \int_{a}^{x} ds \,\delta^{i}(s)\mu^{u}(s)], \tag{16}$$

692 and so

$$e^{i}(a) = \int_{a}^{\infty} dx \, \frac{l^{i}(x)}{l^{i}(a)} \simeq e^{u}(a) - \int_{a}^{\infty} dx \, \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \, \delta^{i}(s) \mu^{u}(s), \quad (17)$$

which implies that the difference in e(a) between state i and the U.S. average is

$$\Delta e(a) = -\int_a^\infty dx \, \frac{l^u(x)}{l^u(a)} \int_a^x ds \, \delta^i(s) \mu^u(s). \tag{18}$$

⁶⁹⁵ Thus the variance of age-at-death after age a for state i is

$$V^{i}(a) = \frac{2}{l^{i}(a)} \int_{a}^{\infty} dx \, x l^{i}(x) - [e^{i}(a)]^{2} - 2ae^{i}(a).$$
(19)

⁶⁹⁶ Denote the terms in the above line as

$$A = \frac{2}{l^{i}(a)} \int_{a}^{\infty} dx \, x l^{i}(x), \ B = [e^{i}(a)]^{2}, \ C = 2ae^{i}(a)$$

⁶⁹⁷ Using our expansions,

$$A = \frac{2}{l^{u}(a)} \int_{a}^{\infty} dx \, x l^{u}(x) - 2 \int_{a}^{\infty} dx \, x \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \, \delta^{i}(s) \mu^{u}(s), \quad (20)$$

698

$$B \simeq [e^{u}(a)]^{2} - 2e^{u}(a) \int_{a}^{\infty} dx \, \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \, \delta^{i}(s) \mu^{u}(s), \qquad (21)$$

699

$$C = 2ae^{u}(a) - 2a \int_{a}^{\infty} dx \, \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \, \delta^{i}(s) \mu^{u}(s)], \tag{22}$$

700 Adding these we find

$$V^{i}(a) = V^{u}(a) + 2\int_{a}^{\infty} dx \left[e^{u}(x) + a - x\right] \frac{l^{u}(x)}{l^{u}(a)} \int_{a}^{x} ds \,\delta^{i}(s)\mu^{u}(s).$$
(23)

The entropy-like quantity and the sensitivity of e_0 and V(a)

The age-pattern of contributions to e_0 gaps in Figure 1b shows that old ages play a major role. That observation marches with a simpler and wellknown analysis: suppose that mortality in state *i* differs from U.S. mortality by the same proportion k^i at every age. Then

$$\delta^{i}(x) = k^{i}, \quad \to \quad e_{0}^{i} = e_{0}^{u} - k^{i} \int_{0}^{\infty} dx \, g^{u}(x),$$
 (24)

where $g^u(x)$ is an entropy-like quantity (Fernandez and Beltrán-Sánchez 2015; Liang, Guo, and Tuljapurkar 2023)

$$g^{u}(x) = -l^{u}(x) \log l^{u}(x).$$
 (25)

Thus a proportional change k^i in mortality at every age changes e_0^u by an amount proportional to $g^u(x)$ at age x (Vaupel 1986; Goldman and Lord 1986; Keyfitz, Caswell et al. 2005). For modern industrialized countries, the entropy-like quantity also shows that old ages matter a great deal (see Figure A-1a).

Figure A-1: The sensitivity of e_0 and V(6) of U.S. females in 1969 and 2019



Note: (a) The sensitivity of e_0^u to proportional changes in $\mu^u(x)$. (b) The sensitivity of $V^u(6)$ to proportional changes in $\mu^u(x)$ *Source*: Authors' calculation based on USMDB (2024).

Similarly, when proportional changes in age-specific mortality rates are uniform across ages, and this results in a change in the variance of age-at-death above age a, given by:

$$\frac{dV(a)}{d\ln\mu(x)} = 2\int_{a}^{\infty} dx \left[e^{u}(x) + a - x\right] h^{u}(x),$$
(26)

where $h^u(x)$ is another entropy-like function, similar in structure to the sensitivity of life expectancy, and defined as:

$$h^{u}(x) = -\frac{l^{u}(x)}{l^{u}(a)} \log \frac{l^{u}(x)}{l^{u}(a)}.$$
(27)

Figure A-1b illustrates the sensitivity of V(a) for U.S. females in 1969 and 2019. Both curves show a "young-old threshold" age T(a): at ages below the threshold, declines in mortality will reduce the variance (by
compressing the distribution of deaths), whereas mortality decline at ages
above the threshold, increase the variance (by widening the distribution of
deaths).

724 Supplementary figures and table

Figure A-2: The mortality gaps between individual states and the U.S. in 1969

(a) Gaps in e_0



(b)

Percentage differences in V(6)





Source: Authors' calculation based on USMDB (2024).

Figure A-3: Mortality dynamics of each state and the U.S., from 1969 to 2019



Note: The changes of mortality indicators here are calculated by values in 2019 minus values in 1969. Each label, the state abbreviations, represents a specific state or the overall US average. The color of each label indicates the direction of changes in V(6): green represents a decrease in V(6), while purple represents an increase in V(6). The magnitude of V(6) changes is reflected in the font size of the state abbreviations. The horizontal axis indicates changes in e_0 , the vertical axis represents changes in the *CDR*.

Source: Authors' calculation based on USMDB (2024).





Note: The shaded regions illustrate the range of mortality indicator values across states, where the upper boundary is the highest observed state-level value and the lower boundary is the lowest one. The solid black line denotes the U.S. national value. *Source*: Authors' calculation based on USMDB (2024).

Figure A-5: The magnitude of contribution from age groups to e_0 gaps between individual states and the U.S. in 1969 and 2019



Note: Different shades indicate the magnitude of contributions to e_0 gaps: slight (<25%), moderate (25%-75%), and strong (>75%).

Source: Authors' calculation based on USMDB (2024).

Figure A-6: The young-old threshold age T(6) of the U.S. from 1969 to 2019



Source: Authors' calculation based on USMDB (2024).

Figure A-7: The percentage differences of V(6) between individual states and the decompositions in 1969 and 2019



Note: The percentage differences in V(6) between individual states and the U.S. are calculated as $[V^i(6) - V^u(6)]/V^u(6)$, and the differences from early and late inequality are also divided by $V^u(6)$. *Source*: Authors' calculation based on USMDB (2024).

Figure A-8: The decomposition for female crude death rate gaps between individual states and the U.S. in 1969 and 2019



Note: Different shades indicate the magnitude of contributions to CDR gaps: slight (<25%), moderate (25%–75%), and strong (>75%). *Source*: Authors' calculation based on USMDB (2024).

Table A-1:	Distribution of the values of $\delta^{i}(x)$ ·	$\mu^u(x)$
Fable A-1:	Distribution of the values of $\delta^{i}(x)$ ·	$\mu^{u}(x)$

	Mean	SD	Minimum	1st percentile	Median	99th percentile	Maximum
Female	-0.00047	0.02080	-0.57795	-0.07157	-0.00003	0.06235	0.29490
Male	-0.00018	0.02235	-0.52204	-0.07806	-0.00003	0.06798	0.35331

Note: Values of $\delta^i(x) \cdot \mu^u(x)$ are calculated across states, years, sexes, and ages. The sex-specific distribution is provided to validate the assumption underlying the linear Taylor expansion, namely that $\delta^i(x) \cdot \mu^u(x) \ll 1$.