The structure of Chinese families who lost their only child: Simulation analysis based on the life-tables

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Introduction

As the number of only children continues to increase, the number of families who have lost their only child is also gradually rising. In 2010, there were 2.41 million families in China that had experienced losing their only child, and there were 0.82 million parents aged 50 and above had lost their only child. The number of mothers aged 50 and above who had lost their only child reached 2.89 million by 2020. The number of families who have lost their only child is predicted to expand at a relatively fast pace in the coming years.

Existing studies have estimated and predicted the number of deceased only children and mothers who losing the only child, and calculated indicators such as the probability of losing child and the survival years of parents after losing their only child. These studies primarily analyze indicators related to losing child from the perspective of either the father or the mother independently. Fewer studies put father, mother, and child together as a whole to examine the structural simulation of families who lost their only child. With the increasing life expectancy and the postponement of marriage and first birth, the structure and life cycle of only-child families have also changed. These factors, along with the age gap between the couple, all influence indicators related to losing the only child.

This paper focuses on families who lost their only child, uses life table and probability method to design formulas for calculating indicators related to losing the only child. In this study, "*shidu* family" refers to a nuclear family consisting of parents and their only child, which develops into a *shidu* family over time due to the death of the only child, without considering the extended family resulting from the child's marriage and reproduction. The main contents of this study are shown as follows: (1) describe age-specific probabilities and cumulative probabilities of 4 different death orders for *shidu* families; (2) analyze the impact parents' age at childbirth on indicators related to losing the only child; (3) explore the effect of the age gap between the couple on these indicators.

Method and Data

Method

In this paper, we incorporate all three members of an only-child family into the analysis, distinguishing the order of death among the family members to simulate the evolution of *shidu* families structural, and calculate the probability of losing the only-child and related indicators. A brief introduction to these formulas is provided below.

The probability of different death orders

Let $l^m(t)$ and $l^f(t)$ represent the the life-table probabilities of surviving from birth to exact age t, where superscripts m and f denote male and female, respectively. the terms $\mu^m(t)$ and $\mu^f(t)$ denote the corresponding risk of dying at age t, where $\mu(t) = -\dfrac{l'(t)}{l(t)}$. It is assumed that a male becomes a mother at the age x, and a female becomes a mother at the age y. As a *shidu* family, the possible order of death include the following four: *child-father-mother*, *child-mother-father*, *mother-child-father*, and *father-child-mother*.

When the order of death is child-father-mother, That is, the only-child dies before reaching age t, and the father dies in the t-th year after the child's birth, the probability of which can be expressed as:

$$P_1 = \int_o^w (1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^m(x+t)dt \qquad (1)$$

Here w is the upper age limit in the definite integral and it is assumed that the maximum age in the life-table is a, so $w = a - \max(x, y)$, w changes with variations in the selected life table and the ages at childbirth. In the actual calculations, the maximum age in the life table we used is 100.

c reprents the child, due to the fact that males and females have different life tables, $\dfrac{l^m(t)}{l^m(0)}$ is used to replace $\dfrac{l^c(t)}{l^c(0)}$ if the child is a son, and $\dfrac{l^f(t)}{l^f(0)}$ is used to replace $\dfrac{l^c(t)}{l^c(0)}$ if the child is a daughter.。

When the order of death is *child-father-mother*, That is, the only-child dies before reaching age t, and the father dies in the t-th year after the child's birth, the probability of which can be expressed as:

The probability of the death order *child-mother-father* is:

$$P_2 = \int_o^w (1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^f(y+t)dt \qquad (2)$$

When the order of death is *father-child-mother*, That is, the father dies before t years after the child's birth, and the only-child dies in the t-th year after birth, the probability of which can be expressed as:

$$P_3 = \int_o^w (1 - \frac{l^m(x+t)}{l^m(x)}) \frac{l^f(y+t)}{l^f(y)} \frac{l^c(t)}{l^c(0)} \mu^c(t)dt \qquad (3)$$

The probability of the death order *mother-child-father* is:

$$P_4 = \int_o^w \frac{l^m(x+t)}{l^m(x)} (1 - \frac{l^f(y+t)}{l^f(y)}) \frac{l^c(t)}{l^c(0)} \mu^c(t)dt \qquad (4)$$

The age at death

Corresponding to Equation (1), if the death order is *child-father-mother*, the age of the father at death can be expressed as:

$$MA_{1,m} = \frac{\int_o^w (x+t)(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)}) \mu^m(x+t)dt}{P_1} \qquad (5)$$

Corresponding to Equation (2), the age of the mother at death can be expressed as:

$$MA_{2,f} = \frac{\int_o^w (y+t)(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)}) \mu^f(y+t)dt}{P_2} \qquad (6)$$

Corresponding to Equation (3), the age of the child at death can be expressed as:

$$MA_{3,c} = \frac{\int_o^w t(1 - \frac{l^m(x+t)}{l^m(x)}) \frac{l^f(y+t)}{l^f(y)} \frac{l^c(t)}{l^c(0)} \mu^c(t)dt}{P_3} \qquad (7)$$

The age of mother at the time of child's death can be expressed as:

$$MA_{3,f} = \frac{\int_o^w (y+t)(1 - \frac{l^m(x+t)}{l^m(x)}) \frac{l^f(y+t)}{l^f(y)} \frac{l^c(t)}{l^c(0)} \mu^c(t)dt}{P_3} \qquad (8)$$

Corresponding to Equation (4), the age of the child at death can be expressed as:

$$MA_{4,c} = \frac{\int_o^w t \frac{l^m(x+t)}{l^m(x)} (1 - \frac{l^f(y+t)}{l^f(y)}) \frac{l^c(t)}{l^c(0)} \mu^c(t)dt}{P_4} \qquad (9)$$

The age of father at the time of child's death can be expressed as:

$$MA_{4,m} = \frac{\int_o^w (x+t) \frac{l^m(x+t)}{l^m(x)} (1 - \frac{l^f(y+t)}{l^f(y)}) \frac{l^c(t)}{l^c(0)} \mu^c(t)dt}{P_4} \tag{10}$$

The duration of marriage

Corresponding to Equation (1), if the death order is *child-father-mother*, and the duration of marriage can be expressed as:

$$MP_{1,m} = \frac{\int_o^w t(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^m(x+t)dt}{P_1} \tag{11}$$

Corresponding to Equation (2), if the death order is *child-mother-father*, the duration of marriage can be expressed as:

$$MP_{2,f} = \frac{\int_o^w t(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^f(y+t)dt}{P_2} \tag{12}$$

Years survived

Corresponding to Equation (1), if the death order is *child-father-mother*, the years that mother survives alone after father's death can be expressed as:

$$MP_{1,f} = \frac{\int_o^w e_{y+t}(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^m(x+t)dt}{P_1} \tag{13}$$

Corresponding to Equation (2), if the death order is *child-mother-father*, the years that father survives alone can be expressed as:

$$MP_{2,m} = \frac{\int_o^w e_{x+t}(1 - \frac{l^c(t)}{l^c(0)}) \frac{l^m(x+t)}{l^m(x)} \frac{l^f(y+t)}{l^f(y)} \mu^f(y+t)dt}{P_2} \tag{14}$$

Corresponding to Equation (3), if the death order is *father-child-mother*, the years that mother survives alone can be expressed as:

$$MP_{3,f} = \frac{\int_o^w e_{y+t}(1 - \frac{l^m(x+t)}{l^m(x)}) \frac{l^f(y+t)}{l^f(y)} \frac{l^c(t)}{l^c(0)} \mu^c(t)dt}{P_3} \tag{15}$$

Corresponding to Equation (4), if the death order is *mother-child-father*, the years that father survives alone can be expressed as:

$$MP_{4,m} = \frac{\int_o^w e_{x+t} \frac{l^m(x+t)}{l^m(x)}(1-\frac{l^f(y+t)}{l^f(y)})\frac{l^c(t)}{l^c(0)}\mu^c(t)dt}{P_4}$$

（16）

Due to the large number of formulas and data results in this paper, the Results and Conclusions section provides calculations and analyses only for cases where the only child is a boy.

Data

According to the above formulas, the data used in this paper includes life table data and age data. The life table data is based on age-specific mortality data in the national population census in 2020, and the death data was adjusted with Bayesian regression model suitable for incomplete death registration. Two gender-specific life tables were constructed in this study.

The ages at childbirth are based on the ages at first marriage. The national population census in 2020 shows that the average age at first marriage for males and females aged 15-49 was 28.30 and 26.91 years, respectively. In this study, it is assumed the age at first marriage for males and females was 29 and 27, respectively. The male becomes a father at age 30 and the female becomes mother at age 28. Based on this, the probabilities of different death orders in *shidu* families are analyzed.

Furthermore, the paper examines the impact of the parents' age at childbirth on indicators related to losing the only-child. It is assumed that the age gap between the couple remains two years, the mother's age at childbirth varies from 20 to 36. In addition, the study also analyzes the impact of the age gap between spouses on *shidu* family indicators. Based on the average age at first marriage by gender in 2020, it is assumed that the mother's age at childbirth remains 28, while the father's age at childbirth varies from 22 to 38.

Results

The age-specific probability of different death orders

Figure 1 shows the age-specific probability of different death orders in *shidu* families, the age-specific probability rises and then drops by child's age. In the case that parents both survive when their only child dies, the age-specific probability of the death order *child-mother-father* peaks at 1.05‰ in the 55th year after the son's birth. If the death order is *child-father-mother*, the probability peaks at 1.56‰ in the 54th year after the son's birth. In the case that only one parent survives when the only-child dies, the probability of the death order *father-child-mother* peaks at 2.40‰ in the 59th year after the son's birth. If the death order is *mother-child-father*, the probability peaks at 0.95‰ in the 58th year after the son's birth.
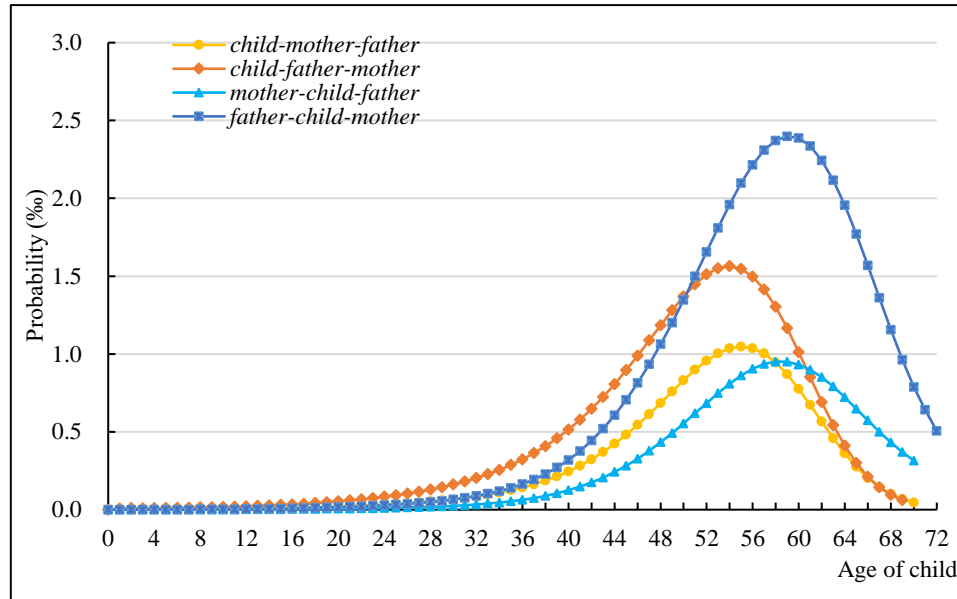
Figure 1. the age-specific probability of different death orders in *shidu* family

The cumulative probability of different death orders

Figure 2 shows the cumulative probability of different death orders in *shidu* families. In the case that both parents survive when their only child dies, the cumulative probability of the death order *child-father-mother* ultimately reaches 3.16%, while the cumulative probability of the death order *child-mother-father* ultimately reaches 1.98%. In the case that only one parent survives when the only-child dies, the cumulative probability of the death order *father-child-mother* ultimately reaches 3.16%, while it ultimately reaches 1.98% for the probability of the death order *mother-child-father*.
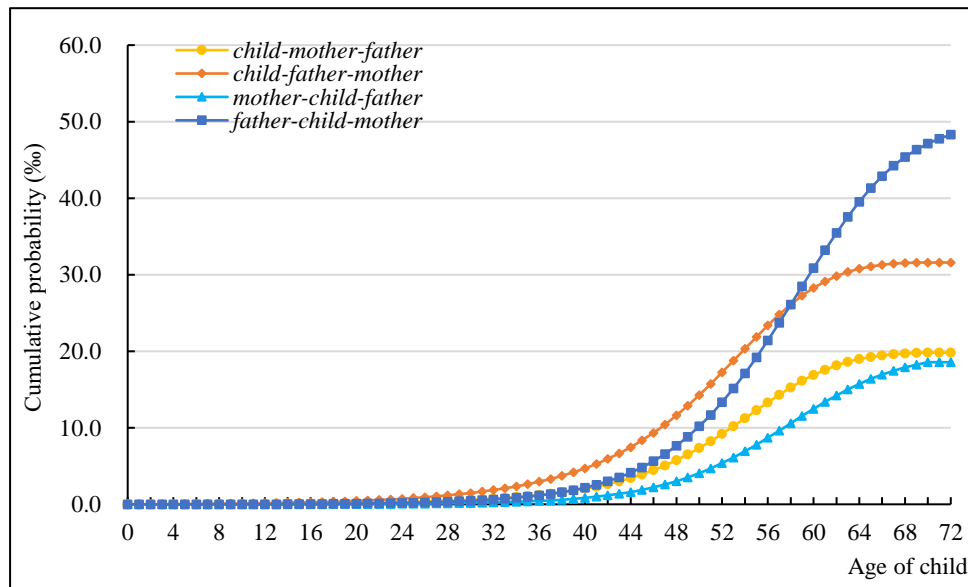

Figure 2. the cumulative probability of different death orders in *shidu* family

The impact of the parents' age at childbirth on the indicators related to *shidu* family

Figure 3 shows the impact of the parents' age at childbirth on the cumulative probability of different death orders in *shidu* families. In the case that both parents survive when their only child dies, as the age at which a female becomes a mother varies from 20 to 36, the cumulative probability of the death order *child-father-mother* decreases from 5.42% to 1.97%, and it decreases from 3.43% to 1.22% for probability of the death order *child-mother-father*. In the case that only one parent survives when the only-child dies, as mother's age at childbirth varies from 20 to 36, the cumulative probability of the death order *father-child-mother* decreases from 8.59% to 2.46%, and it decreases from 3.44% to 0.97% for the cumulative probability of the death order *mother-child-father*.
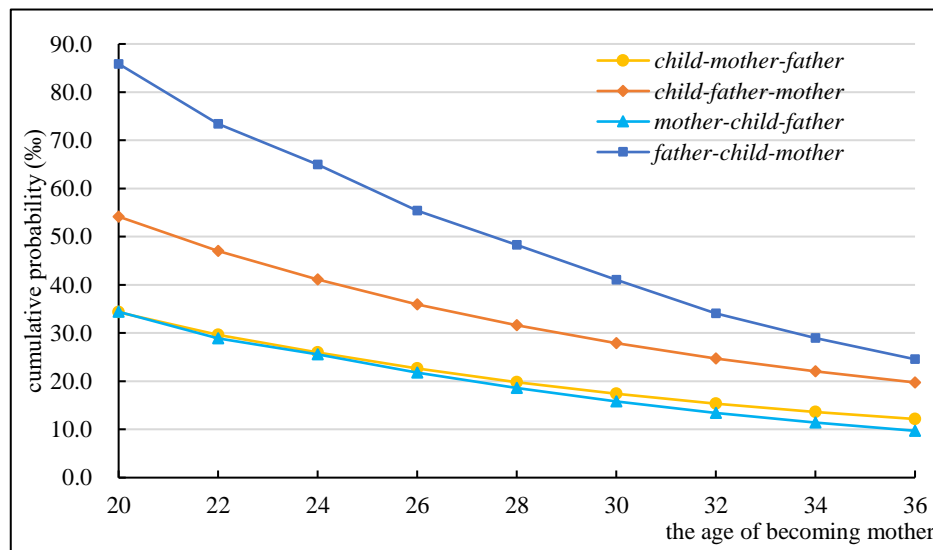

Figure 3. the impact of the parents' age at childbirth on the cumulative probability of different death orders in *shidu* families

Table 1 shows the impact of the parents' age at childbirth on the indicators related to shidu family in the case that both parents survive when their only child dies. If the death order is *child-father-mother*, as the age at which a female becomes a mother varies from 20 to 36, the duration of the parents' marriage decreases from 58.29 years to 41.56 years, the age of father at death decreases from 80.29 to 79.56, while the years that mother survives alone are with no significant change. If the death order is *child-mother-father*, as the mother's age at childbirth varies from 20 to 36, the duration of the parents' marriage decreases from 60.82 years to 43.60 years, the age of mother at death decreases from 80.82 to 79.60, and the years that father survives alone increase from 7.91years to 8.55 years.

Table 1. the impact of parents' age at childbirth on the indicators related to *shidu* family in the case that both parents survive when their only child dies

Parents' age at childbirth		Death order: *child-father-mother*				Death order: *child-mothe- father*			
Father	Father	cumulative probability (%)	the duration of marriage	the age of father at death	The years that mother survives alone	cumulative probability (%)	the duration of marriage	the age of mother at death	The years that father survives alone
22	20	5.42	58.29	80.29	10.55	3.43	60.82	80.82	7.91
24	22	4.70	56.88	80.88	10.70	2.97	58.65	80.65	7.99
26	24	4.11	54.76	80.76	10.79	2.60	56.63	80.63	8.03
28	26	3.60	52.59	80.59	10.91	2.26	54.49	80.49	8.10
30	28	3.16	50.40	80.40	11.03	1.98	52.34	80.34	8.18
32	30	2.79	48.20	80.20	11.18	1.74	50.17	80.17	8.27
34	32	2.47	45.99	79.99	11.32	1.54	47.99	79.99	8.36
36	34	2.20	43.77	79.77	11.46	1.36	45.80	79.80	8.45
38	36	1.97	41.56	79.56	11.59	1.22	43.60	79.60	8.55

Table 2 shows the impact of parents' age at childbirth on the indicators related to *shidu* family in the case that only one parent survives when the only-child dies. If the death order is *mother-child-father*, as the mother's age at childbirth varies from 20 to 36, the age of son at death decreases from 64.20 to 48.76, the years that father survives alone are with no significant change. If the death order is *father-child-mother*, as the mother's age at childbirth varies from 20 to 36, the age of son at death decreases from 64.14 to 48.93, and the years that mother survives alone are with no significant change.

Table 2. the impact of age of becoming parents on the indicators related to lost-only-child family in the case that only one parent survives when their only child dies

Parents' age at childbirth		Death order: *mother-child-father*				Death order: *father-child-mother*			
Father	Mother	cumulative probability (%)	Age of son at death	the age of father at the time of child's death	The years that father survives alone	cumulative probability (%)	Age of son at death	the age of mother at the time of child's death	The years that mother survives alone
22	20	3.44	64.20	86.20	6.68	8.59	64.14	84.14	7.81
24	22	2.89	62.02	86.02	6.71	7.34	62.20	84.20	7.78
26	24	2.56	60.56	86.56	6.60	6.50	60.74	84.74	7.60
28	26	2.18	58.61	86.61	6.58	5.54	58.78	84.78	7.58
30	28	1.86	56.65	86.65	6.57	4.71	56.82	84.82	7.56
32	30	1.58	54.68	86.68	6.55	4.01	54.85	84.85	7.54
34	32	1.34	52.71	86.71	6.54	3.41	52.88	84.88	7.53
36	34	1.14	50.74	86.74	6.54	2.89	50.91	84.91	7.52
38	36	0.97	48.76	86.76	6.53	2.46	48.93	84.93	7.51

The impact of the age gap between the couple on the indicators related to shidu family

Figure 4 shows the impact of the age gap between the couple on the cumulative

probability of different death orders in *shidu* families. In the case that both parents survive when their only child dies, as the age gap between the couple varies from -6 to 10, the cumulative probability of the death order *child-father-mother* decreases from 3.89% to 0.76%, and it is with no significant change for the cumulative probability of the death order *child-mother-father*. In the case that only one parent survives when the only-child dies, as the age gap between the couple varies from -6 to 10, the cumulative probability of the death order *mother-child-father* decreases from 5.14% to 0.50%, and it increases from 3.16% to 5.04% for the cumulative probability of the death order *father-child-mother*.
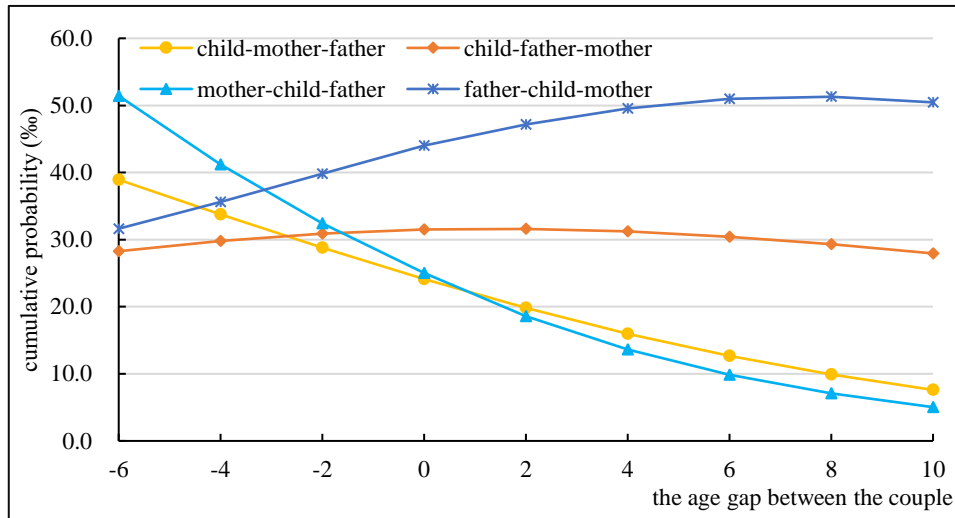

Figure 4. the impact of the age gap between the couple on the cumulative probability of different death orders in *shidu* families.

Table 3 shows the impact of the age gap between the couple on the indicators related to *shidu* family in the case that both parents survive when their only child dies. If the death order is *child-father-mother*, as age gap between the couple varies from -6 to 10, the duration of the parents' marriage decreases from 54.73 years to 41.56 years, the age of father at death increases from 76.73 to 82.21, while the period that mother survives alone increase from 8.75 years to 15.04 years. If the death order is *child-mother-father*, as age gap between the couple varies from -6 to 10, the duration of the parents' marriage decreases from 55.85 years to 46.78 years, the age of mother at death decreases from 83.85 to 74.78, and the period that father survives alone decreases from 10.09 years to 7.45 years.

Table 3. the impact of age gap between the couple on the indicators related to *shidu* family in the case that both parents survive when their only child dies

Parents' age at childbirth		Death order: *child-father-mother*				Death order: *child-mothe- father*			
Father	Father	cumulative probability (%)	the duration of marriage	the age of father at death）	The years that mother survives alone	cumulative probability (%)	the duration of marriage	the age of mother at death）	The years that father survives alone
22	28	2.83	54.73	76.73	8.75	3.89	55.85	83.85	10.09
24	28	2.98	53.83	77.83	9.19	3.38	55.13	83.13	9.49
26	28	3.09	52.82	78.82	9.71	2.88	54.30	82.30	8.97
28	28	3.15	51.67	79.67	10.32	2.41	53.38	81.38	8.54
30	28	3.16	50.40	80.40	11.04	1.98	52.34	80.34	8.18
32	28	3.12	49.02	81.02	11.86	1.60	51.15	79.15	7.90
34	28	3.04	47.52	81.52	12.80	1.27	49.84	77.84	7.69
36	28	2.93	45.91	81.91	13.86	0.99	48.38	76.38	7.54
38	28	2.79	44.21	82.21	15.04	0.76	46.78	74.78	7.45

As table 4 shows, if the death order is *mother-child-father*, as age gap between the couple varies from -6 to 10, the age of son at death decreases from 61.22 to 49.79, the period that father survives alone is with no significant change. If the death order is *father-child-mother*, as age gap between the couple varies from -6 to 10, the age of son at death decreases from 58.51 to 52.47, and the period that mother survives alone increase from 6.89 years to 9.52 years.

Table 4 the impact of age gap between the couple on the indicators related to *shidu* family in the case that only one parent survives when their only child dies

Parents' age at childbirth		Death order: *mother-child-father*				Death order: *father-child-mother*			
Father	Mother	cumulative probability (%)	Age of son at death	the age of father at the time of child's death	The years that father survives alone	cumulative probability (%)	Age of son at death	the age of mother at the time of child's death	The years that mother survives alone
22	28	5.14	61.22	83.22	7.58	3.16	58.51	86.51	6.89
24	28	4.12	60.31	84.31	7.22	3.56	58.23	86.23	7.00
26	28	3.24	59.30	85.30	6.93	3.98	57.92	85.92	7.13
28	28	2.50	58.21	86.21	6.70	4.40	57.56	85.56	7.28
30	28	1.86	56.65	86.65	6.57	4.71	56.82	84.82	7.56
32	28	1.36	55.03	87.03	6.46	4.95	55.94	83.94	7.91
34	28	0.99	53.35	87.35	6.38	5.10	54.93	82.93	8.34
36	28	0.71	51.60	87.60	6.32	5.13	53.77	81.77	8.87
38	28	0.50	49.79	87.79	6.28	5.04	52.47	80.47	9.52

Conclusions

Based on the national population census in 2000 in China, we use life tables and probability method to analyse the indices related to the structure of *shidu* family in different death orders, the results show:

The probability of 4 different death orders for lost-only-child families all tend to rise and then drop by age, and the 54th to 59th year after the child birth is the peak period when the *shidu* family turns into a single-parent family.

The cumulative probability of every death orders climbs with the years after the birth of the son, and which at the maximum age of death in order *father-son-mother* is the highest among all death orders.

The parents' age at childbirth has impact on indicators related to losing the only child. Firstly, the cumulative probability of every different death order in *shidu* families decreases as the age of becoming parents increases. Secondly, the older parents are when child birth, the younger the child tends to be at death. Lastly, the parents' age at childbirth has no impact on the age of parents at child's death and the period of parents surviving alone. Besides, there is no significant difference between two death orders for parents. While, the reduction in the duration of marriage among *shidu* parents is directly related to postponement of first marriage.

The age gap between the couple has impact on indicators related to losing the only child. Firstly, the cumulative probability of the death order that mother and child die before father decreases as the age gap between the couple increases. Secondly, older the parents are at the child's birth, younger the child is at his death. In addition, larger age gap between the couple leads to a shorter duration of marriage, with no significant difference between two death orders for parents. Besides, as the age gap between couple increases, the age of father at the time of child's death increases, while the age of mother decreases. And it results in a larger probability of *shidu* families evolving into single-mother families, with the period that mother survives alone longer.