The Stationary Population Model and Its Identity: A Unifying Framework for Global Aging

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THEORETICAL FOCUS

This paper introduces a novel approach to understanding population aging inspired by the stationary population identity—an indisputable truth in mathematical demography that equates the distributions of life lived and life left in replacement populations. Leveraging this core concept, the paper develops a model that we refer to as the Paired Population Analysis, a comparative framework that juxtaposes the stationary population model, representing a hypothetical zero-growth population, with actual observed populations. This comparison generates new comparative metrics, such as the Dual Average Age, Dissimilarity Indices and Dynamic Age Index, which provide a deeper understanding of population aging by revealing the alignment or divergence between idealized stationary models and real-world dynamics.

DATA SOURCE

The study relies on two complementary datasets from the United Nations World Population Prospects (2022). These datasets cover population data for both sexes combined from 1950 to 2100. The first dataset provides the number of individuals by age for each country and year, while the second dataset contains life table information, including age-specific survival rates, mortality rates, and life expectancy at birth. These data are essential for generating both the stationary population models and the observed population data, which form the basis of the comparative analyses in the study.

RESEARCH METHODS

Guided by the Stationary Population Identity, we introduce the Paired Population Analysis, which juxtaposes the stationary population model—a hypothetical zero-growth population—against the actual observed population from which the life table data are derived. This model parallels experimental methods, where control and treatment groups or matched pairs are either analyzed side by side or the control is used as a frame-of-reference for the experimental group to uncover significant differences and patterns. The framework offers a comparative approach to understanding population aging by highlighting differences between the idealized scenario of stationarity and the complexities of real-world population dynamics. This approach not only provides deeper insights into demographic aging but also facilitates the creation of new metrics for comparative analysis.

MAIN FINDINGS

The study's global analysis reveals several significant trends in population aging. One key finding is the convergence of regional populations toward an average population age-to-average years remaining ratio of unity by 2100. This convergence indicates that many global populations are moving toward a balance between life lived and life left, reflecting broader trends in increasing life expectancy and declining fertility. Regions such as Europe and North America are expected to reach this demographic crossover earlier, while regions like Africa may experience this transition later due to differing demographic dynamics, including higher fertility rates and lower life expectancy. Ten specific findings include the following, the focus of which is on metrics or methods that differ from traditional approaches involving population aging.

METHODS: (1) Stationary Population Identity: This concept equates the fraction of a population at a given age with those having the same years remaining, shifting focus from counting individuals in age brackets to understanding the balance between life lived and life left. (2) Birth-Death Cohort Graphs (Fig 1): These graphs depict birth and death cohort distributions, highlighting demographic trends and aging processes more comprehensively than traditional population pyramids or static age metrics like % 65. (3) Population Death Heat Maps (Fig. 2): Visualizing death distributions across birth cohorts over time, these heat maps offer dynamic insights into demographic aging patterns, providing a more detailed analysis than static age-based metrics like % 65 or OADR. (4) Paired Population Set (PPS): PPS is a comparative framework contrasting the stationary population model with observed populations, enabling direct comparison between idealized scenarios and real-world dynamics, unlike conventional metrics focused on static age groups like % 65.

METRICS: (1) Dual Average Age ($\overline{D}$): $\overline{D}$ combines chronological age (life lived) and thanatological age (life left), providing a comprehensive view of aging by integrating both aspects, offering deeper insights into aging beyond traditional metrics like % 65 or OAD (Fig. 5)R. (2) Dissimilarity Indices (Fig. 8): These indices quantify the divergence between an observed population's age structure and a stationary counterpart, offering a precise measure of how populations deviate from ideal demographic conditions, unlike traditional static metrics. (3) Structural Stationarity: This concept expands population stationarity beyond zero growth and/or Total Fertility Rate of $\leq$ 2.1, emphasizing the balance between years lived and years remaining, introducing a new dimension to population aging analysis that traditional metrics do not address. (4) Dynamic Aging Index (DAI): DAI measures the relationship between changes in average population age and average years remaining, offering a dynamic view of aging that considers improvements in health, unlike static metrics that focus on economic burden (Fig. 7)s. (5) Pinch Points in Population Growth: These thresholds identify transitions from population growth to decline relative to age structure, providing a nuanced understanding of demographic changes that static metrics like % 65 or OADR cannot capture (Fig. 8). (6) Dual Age Ratio (DAR): When DAR equals unity (DAR = 1.00), it indicates that the population is in a state of structural stationarity, where the mean age of individuals in the population equals the mean number of years remaining (Fig. 6).

Overall, the study presents a novel approach to understanding population aging by leveraging the stationary population identity as a theoretical foundation. Through its innovative use of comparative metrics and visual tools, the research offers new insights into global demographic transitions and highlights the importance of considering both life lived and life left in analyses of population aging.

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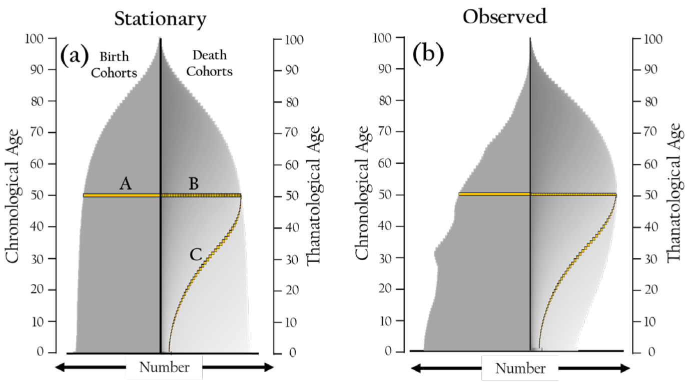

Figures 1a and 1b. Illustrate the distribution of birth and death cohorts ordered by chronological and thanatological ages for the 2020 observed world population and its hypothetical stationary counterpart. Each figure features a birth-death cohort graph with two panels: the left shows the distribution of individuals by age, and the right shows projected deaths over time. In Figure 1a (Stationary Population), the distributions are symmetrical, making Bar A, Bar B, and Diagonal Band C equal at approximately 95 million. This reflects a balanced age structure where birth and death cohort distributions match. In Figure 1b (Observed Population), Bar A′ (approximately 92 million) at age 50 represents individuals living at that age. Bar B′ (approximately 110 million) is a stacked bar representing individuals with at least 50 years remaining. Diagonal Band C′ (approximately 92 million) extends from Bar A to age 100, representing projected deaths for the 50-year-olds.

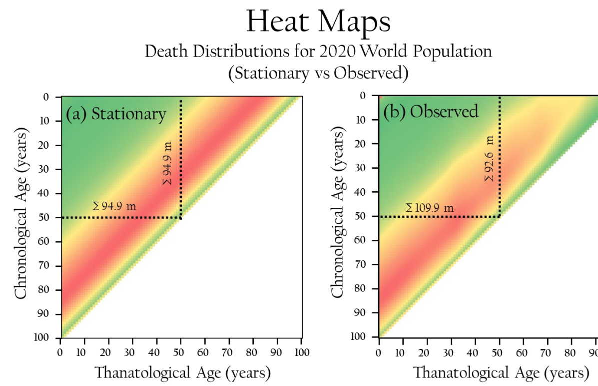

Figure 2. Heat maps of upper traiangular matrices for the standing 2020 world population. Each graphic displays a 101 x 101 square matrix. In the first (left-most) column, the number of deaths by age are color-coded in cohorts aged 0 to 100 (top to bottom) at time 0. Deaths from each birth cohort are tracked across each row from left to right as the cohort ages until extinction at age 100 years. (a) **Stationary**: This map displays the distribution of remaining lives using data from the 2020 period life table for the world. In a stationary population, the sums of rows and columns are equal (e.g., sum of 50 year row equals sum of 50 year column), and all elements along each anti-diagonal are identical. (b) **Observed**: This map shows the distribution of deaths based on calculations from the period life table and estimated cohort sizes. In this non-stationary population, neither the row and column sums nor the anti-diagonal elements are equal (e.g., sum of 50 year row and sum of 50 year column are not equal).

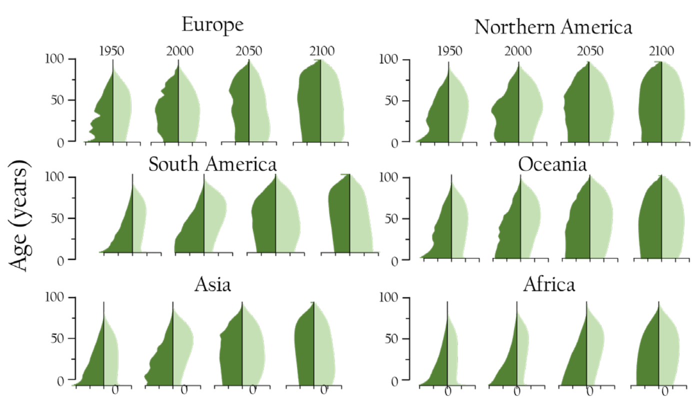

Figure 3. Years lived (left panels) and years left (right panels) distributions for six world regions: Europe, Northern America, South America, Oceania, Asia and Africa for 1950, 2000, 2050 and 2100. Each graphic is normalized to show the general trends of age structure from younger to older populations and for years remaining from more to fewer. Note the similarity in distributions across all world regions in 2100.

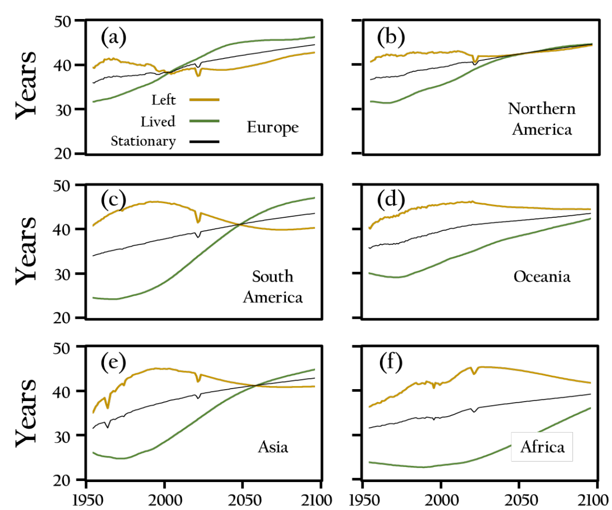

Figure 4. Trajectories for years lived, years remaining, and life table stationary age across six world regions from 1950 to 2100. Note the convergence of years lived and years remaining in all regions, including crossovers in Europe, South America, and Asia.

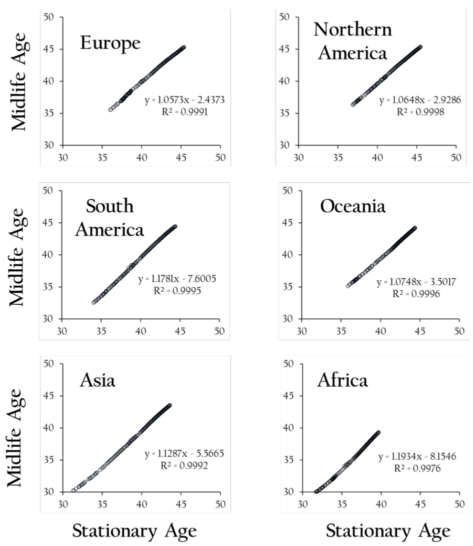

Figure 5. Relationship of Stationary Age and Dual Average Age ($\bar{D}$) for the populations of six world regions in 2000, the former defined as the average age of the stationary population and the latter as the average of the means of years lived and years remaining.

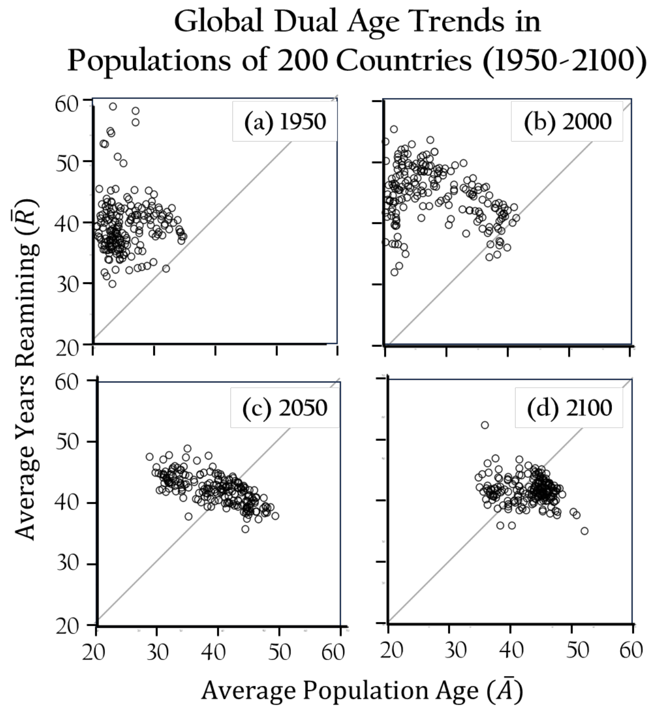

Figure 6. Plots of average years lived versus average number of years remaining for the populations of 200 countries in 1950, 2000, 2050 and 2100—i.e., the Dual Age Ratio (DAR). The number and percent of countries whose average population age was greater than the average years remaining was 1 (0.5%), 9 (4.5%), 73 (36.5%) and 136 (68%) for 1950, 2000, 2050 and 2100, respectively.

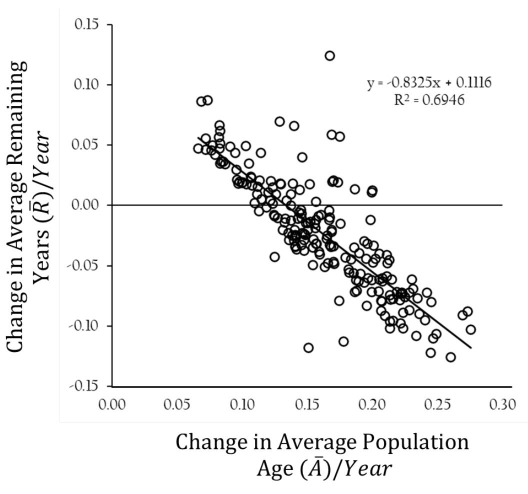

Figure 7. Relationship between the means of change in average population age and of average remaining years in the populations of 200 countries from 2000 to 2100. Note the value of the slope of the regression indicating a change of 0.8325 years in the mean number of years remaining for every one year change in the mean population ages—i.e. the Dynamic Aging Index (DAI)

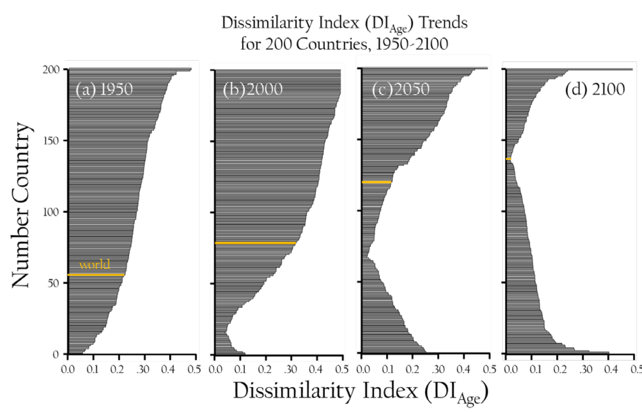

Figure 8. Horizonal bar graph distributions of dissimilarity indices comparing the mean ages of the observed populations with their stationary complements, DI_{Age}, for 200 countries and the world population (gold-shaded bar) in four years from 1950 through 2100. The narrowest portion (shortest bars) of the distributions coincide with the change-over from positive population growth (above) to negative (below).