Growing Old Together? The Sex Gap in Population Ageing

Tabitha Scott and Vladimir Canudas-Romo

Abstract

Population ageing is one of the most significant population phenomena of this century. Across time and populations, females are, on average, older than males, a trend that is underexplored in the literature. This study presents the sex gap in population ageing, measured with the population mean age, and seeks to understand how it is related to sex differences in mortality. A new novel measure of the age distribution of mortality is presented, termed the 'life table mean age'. The sex gap in the life table mean age is then compared with the sex gap in the population mean age, revealing a very similar time trend. Further, age-specific decomposition of the population mean age and life table mean age reveal that it is largely sex differences above the age of 60 that produce the sex gap in both means. Looking at the variable-r decomposition of changes in the population mean age shows that historical changes in survivorship are the main contributor in most studied populations. This study contributes to the literature by showing that the sex gap in population ageing can be accounted for by sex differences in mortality at older ages.

Motivation

Population ageing is one of the most significant population phenomena of this century. Population ageing is far from uniform, affecting subpopulations in a variety of ways. One such example is population ageing by sex. Across time and populations, females are, on average, older than males, a trend that is underexplored in the literature. The sex gap in ageing can be seen in the number of female centenarians who, in 2015, outnumbered male centenarians of almost 4-to-1 (Robine & Cubaynes 2017). In considering centenarians, the relationship between population ageing and another demographic factor, increasing life expectancies, can be considered.

Population ageing and life expectancy are closely related; increasing life expectancies is a key contributor to population ageing. Female life expectancies exceed male life expectancies throughout the world (United Nations 2022, p. 18). Little is known, however, about the extent to which this sex gap in mortality influences the sex gap in ageing.

Theoretical focus and research question

This study, utilising a mathematical demographic approach, seeks to understand how the sex gap in population ageing, measured with the population mean age, is related to sex differences in mortality.

Data

This study analyses data from 13 countries, Norway, Switzerland, England and Wales, Scotland, the Netherlands, Sweden, Finland, Denmark, France, Spain, Italy, Australia and the US, but focuses mainly on Spain, Sweden, the US and Australia. The analysis is based on population counts and life expectancy data, sourced from the Human Mortality Database (HMD 2024) and utilises the complete time span of data available on the HMD for each country.

Methods

Population mean age

Let $\bar{a}$(t) denote the population mean age at time t. This is defined as

$$\bar{a}(t) = \frac{\int_0^\omega a\, N(a,t)\, da}{N(t)}, \qquad (1)$$

where $N(a,t)$ is the number of people aged a at time t, $N(t)$ is the total number of people at all ages at time t, or $N(t) = \int_0^\omega N(a,t)\, da$, and ω represents the maximum age in the population.

The sex gap in the population mean age

Let $\bar{a}_F(t)$ and $\bar{a}_M(t)$ represent the mean age for the females and males in a population, respectively. The sex gap in population mean age will be denoted as $\Delta\bar{a}(t) = \bar{a}_F(t) - \bar{a}_M(t)$. Using the definition of the population mean age given in Eq(1) and rearranging gives an age-specific expression of the sex gap in population mean age:

$$\Delta\bar{a}(t) = \int_0^\omega a[c_F(a,t) - c_M(a,t)]\, da, \qquad (2)$$

where $c_i(a,t)$ is the proportion of the population at age a, defined as $c_i(a,t) = \frac{N_i(a,t)}{N_i(t)}$ at time t, and this is restricted to the female or male subset of the population as indicted with subscript i.

The change in population mean age with respect to time

Let a dot above a variable represent the derivative with respect to time (Vaupel and Canudas-Romo 2002). The change in the population mean age with respect to time can be denoted $\dot{\bar{a}}(t)$. Preston, Himes and Eggers (1989) showed that the age-decomposition of the change in population mean age over time can be expressed as:

$$\dot{\bar{a}}(t) = \int_0^\omega r(a,t)c(a,t)[a - \bar{a}(t)]\, da, \qquad (3)$$

where $r(a,t)$ is the growth rate of the population at age a, defined as the relative change in the population counts or $r(a,t) = \frac{\dot{N}(a,t)}{N(a,t)}$.

Variable-r decomposition of the change in population mean age with respect to time

The growth rate of a population can be expressed as the sum of the growth rate at birth (r_B), changes in survivorship (Δs) and changes in migration (Δm) (Horiuchi & Preston 1988):

$$r(a,t) = r_b(t-a) + \Delta s(a,t) + \Delta m(a,t). \qquad (4)$$

When Eq(4) is substituted into Eq(3) and the result is rearranged, the change in population mean age can be decomposed into the contributions of changes in births, survivorship and migration (variable-*r* decomposition), or expressed as

$$\dot{\bar{a}}(t) = \widetilde{r_b} + \widetilde{\Delta s} + \widetilde{\Delta m}, \quad (5)$$

where the tilde above the terms of Eq(4), for example $\widetilde{r_b}$, correspond to the right side of Eq(3), but substituting $r(a,t)$ for each of the terms in Eq(4), e.g. $r_b(t-a)$, and where the weight includes the population composition and the difference of age a to the mean age $c(a,t)[a-\bar{a}(t)]$ as seen in Eq(3).

The change over time in the sex gap of the population mean age

It is also possible to study the change in population mean age over time and also by sex – similar to the "contour decomposition" approach (Jdanov et al. 2017).

Let $\dot{\bar{a}}_F(t)$ and $\dot{\bar{a}}_M(t)$ represent the change over time of mean age for females and males in a population, respectively. Denote their difference, which is the change over time in the sex gap of the population mean age, as $\Delta\dot{\bar{a}}(t) = \dot{\bar{a}}_F(t) - \dot{\bar{a}}_M(t)$. Applying Eq(2) to each of $\dot{\bar{a}}_F(t)$ and $\dot{\bar{a}}_M(t)$ produces

$$\Delta\dot{\bar{a}}(t) = \int_0^{\omega} r_F(a,t)c_F(a,t)[a-\bar{a}_F(t)]\, da - \int_0^{\omega} r_M(a,t)c_M(a,t)[a-\bar{a}_M(t)]\, da. \quad (6)$$

A further variable-r decomposition substituting Eq(4) into the female and male components of Eq(6) can be used to disentangle the contribution of historical changes in births, survivorship and migration to the change over time in the sex gap of the population mean age.

Life table mean age

The life table mean age, denoted $\bar{a}_l(t)$, is the average age of survivors in a life table. It is defined as

$$\bar{a}_l(t) = \frac{\int_0^{\omega} a\, l(a,t)\, da}{e(t)}, \quad (7)$$

where $l(a,t)$ is the life table survivors at age a, with radix equal to one, or $l(0,t) = 1$, and the life expectancy at birth in that life table is $e(t) = \int_0^{\omega} l(a,t)\, da$, all calculated at time t. The maximum age in the life table is represented by ω.

The change in life table mean age

Careful inspection of Eq(1) and Eq(7) reveal that the life table mean age is identical to the population mean age, with the only difference being that $N(a,t)$, or the number of people in a population aged a at time t in Eq(1) is replaced with $l(a,t)$, or the number of survivors in the life table aged a at time t in Eq(7). Therefore, by replacing $N(a,t)$ with $l(a,t)$ where necessary, the change in the life table mean age with respect to time, the sex gap in the life table mean age and the change over time in the sex gap of the life table mean age can be expressed in ways analogous to those for the population mean age.

The sex gap in the life table mean age can be expressed as

$$\Delta\bar{a}_l(t) = \int_0^{\omega} a\left[c_{l,F}(a,t) - c_{l,M}(a,t)\right] da \, , \qquad (8)$$

where $c_{l,i}(a,t)$ is the proportion of the life table survivors at age a, or the contribution of age a to life expectancy, defined as $c_{l,i}(a,t) = \frac{l(a,t)}{e(t)}$, and this is restricted to the female or male subset of the population as indicted by subscript i.

The change in life table mean age with respect to time can be expressed as

$$\dot{\bar{a}}_l(t) = \int_0^{\omega} r_l(a,t) c_l(a,t)[a - \bar{a}_l(t)] \, da, \qquad (9)$$

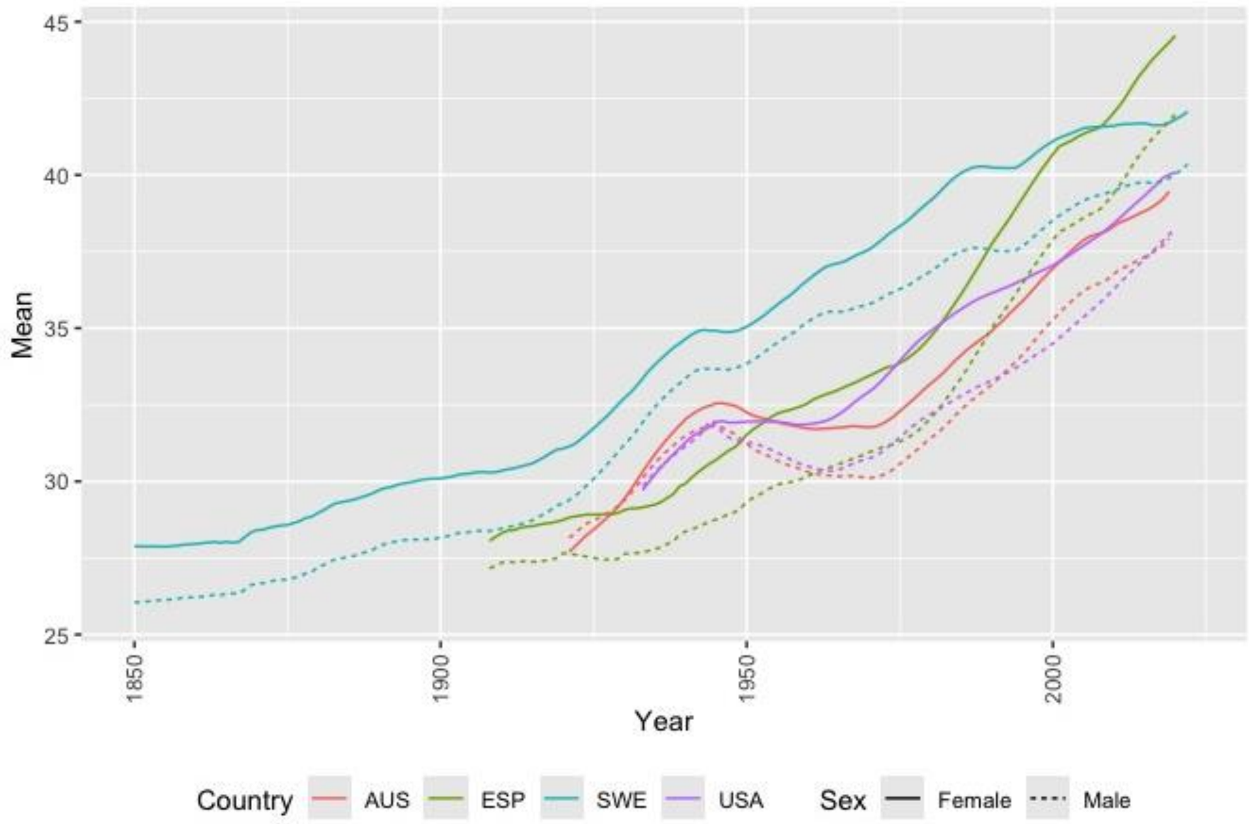
where $r_l(a,t)$ is the growth rate in the life table survivors at age a, or relative change over time, defined as $r_l(a,t) = \frac{\dot{l}(a,t)}{l(a,t)}$.

The sex gap in the change in life table mean age with respect to time can be expressed as $\Delta\dot{\bar{a}}_l(t) = \dot{\bar{a}}_{l,F}(t) - \dot{\bar{a}}_{l,M}(t)$, and Eq(9), applied to the female or male subset of the population, can be substituted to represent age-specific contributions.

Results

Figure 1 shows the population mean age over time and by sex for Australia, Spain, Sweden and the United States. The population mean ages for males and females have increased over time, with a slight decline in the 1960s for both sexes in Australian and males in the United States. The figure shows that aside from a few years early in the Australian and United States series, the female population mean age is consistently above male population mean age over time for all populations.

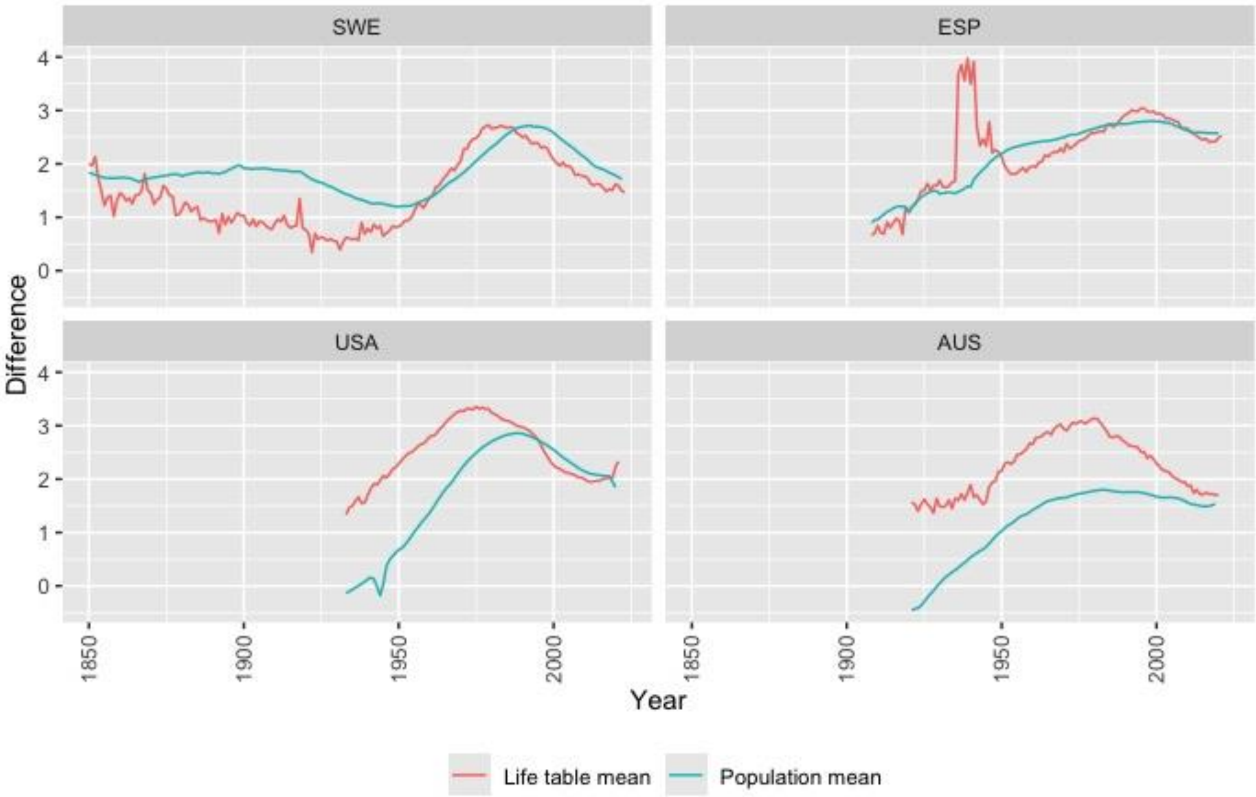
Figure 1: Population mean age by year, sex and country



Source: Human Mortality Database 2024

Figure 2 introduces the life table mean age, showing the sex gap in the population mean age (Eq. (2)) and the life table mean age (Eq. (8)) over time for Australia, Spain, Sweden and the United States. Consistent with Figure 1, aside from the early Australian and United States series, the sex gap in the population mean age is entirely positive, meaning that the female population mean age is higher than the male population mean age. For the life table mean age, the sex gap is exclusively in favour of women. It can be seen that the sex gap in both means follow a similar pattern over time, though in the Unites States and Australia the sex gap for the population mean age is typically lower than the sex gap for the life table mean age. The four populations follow a similar general pattern of an increasing sex gap in both means from the 1950s or earlier which culminates in a peak of around 3 years in the 1980s before declining. Spain contributes some exceptions to this pattern, namely the sex gap in the life table mean age spikes around the late 1930s to early 1940s and the sex gap in the population mean age has remained fairly constant, sitting at around 3 years since the 1970s.

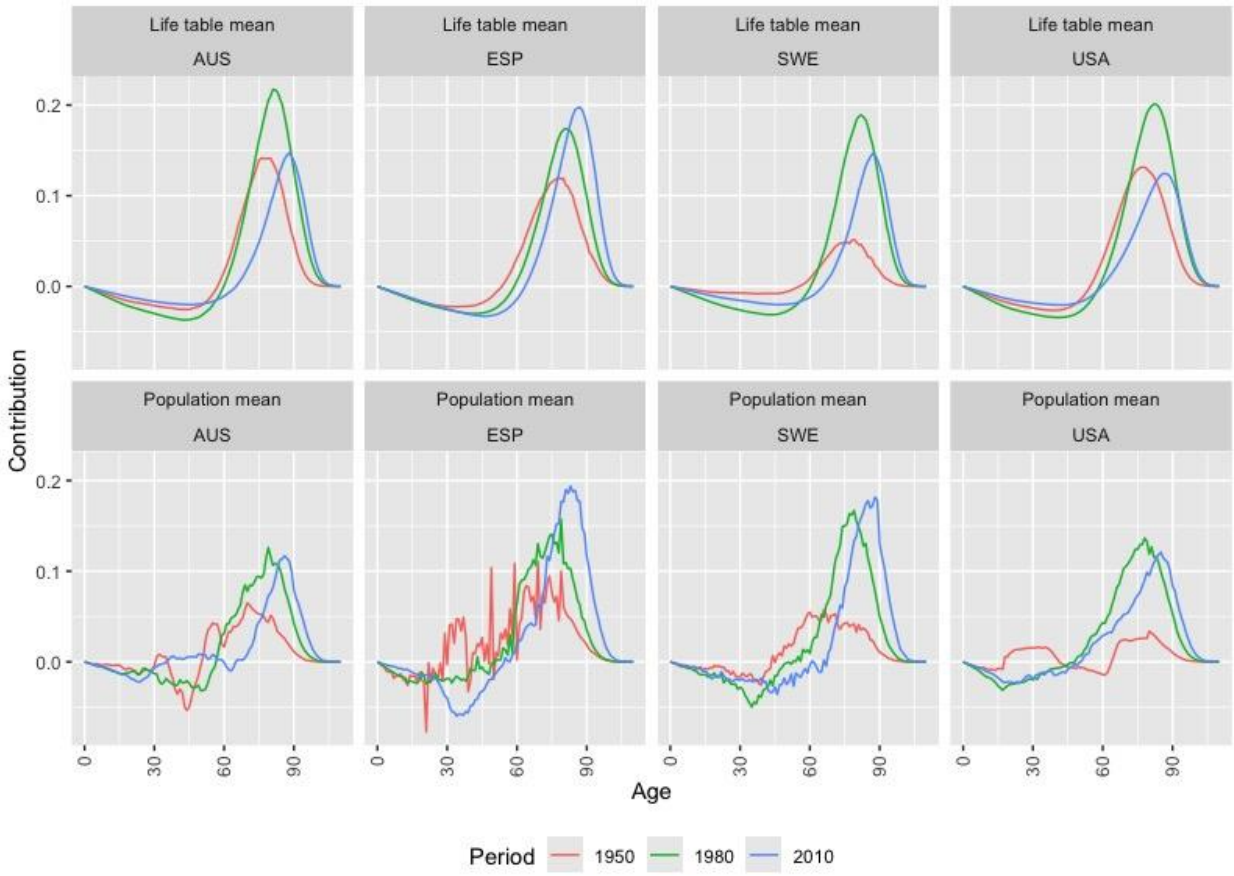
Figure 2: Sex gap in population and life table mean ages by year and country



Source: Human Mortality Database 2024

Figure 3 shows age-specific contributions to the sex gap in both means (Eq. (2) and Eq. (8)) for Australia, Spain, Sweden and the United States at three time points: 1950, 1980 and 2010. With the exception of Spain, there is typically a greater contribution to the sex gap in 1980 than in the other two years. This result is consistent with the peak in the sex gap in both means in the 1980s seen in Figure 2 in all countries except Spain. Although there is more fluctuation in the sex gap for the population mean age than the life table mean age, both sex gaps follow a similar age pattern. The majority of the age-specific contribution occurs above the age of 60, though this result is stronger for the sex gap in the life table mean age than for the population mean age. For all populations and time points for the life table mean age and some for the population mean age, the sex gap below the age of 60 is small but negative, meaning that these ages contribute to a lower overall sex gap. This corresponds to a greater age-specific proportion of males than females for the gap in population mean age, and a higher ratio of male survivors to their life expectancy than female survivors to their life expectancy for the gap in the life table mean age. Figure 2 shows that the sex gap in the life table mean age widens pre-1980 and narrows post-1980. Figure 3 expands on this by showing that both the widening and narrowing of this gap are almost entirely explained by changes above the age of 60. This result is present but weaker for the population mean age.

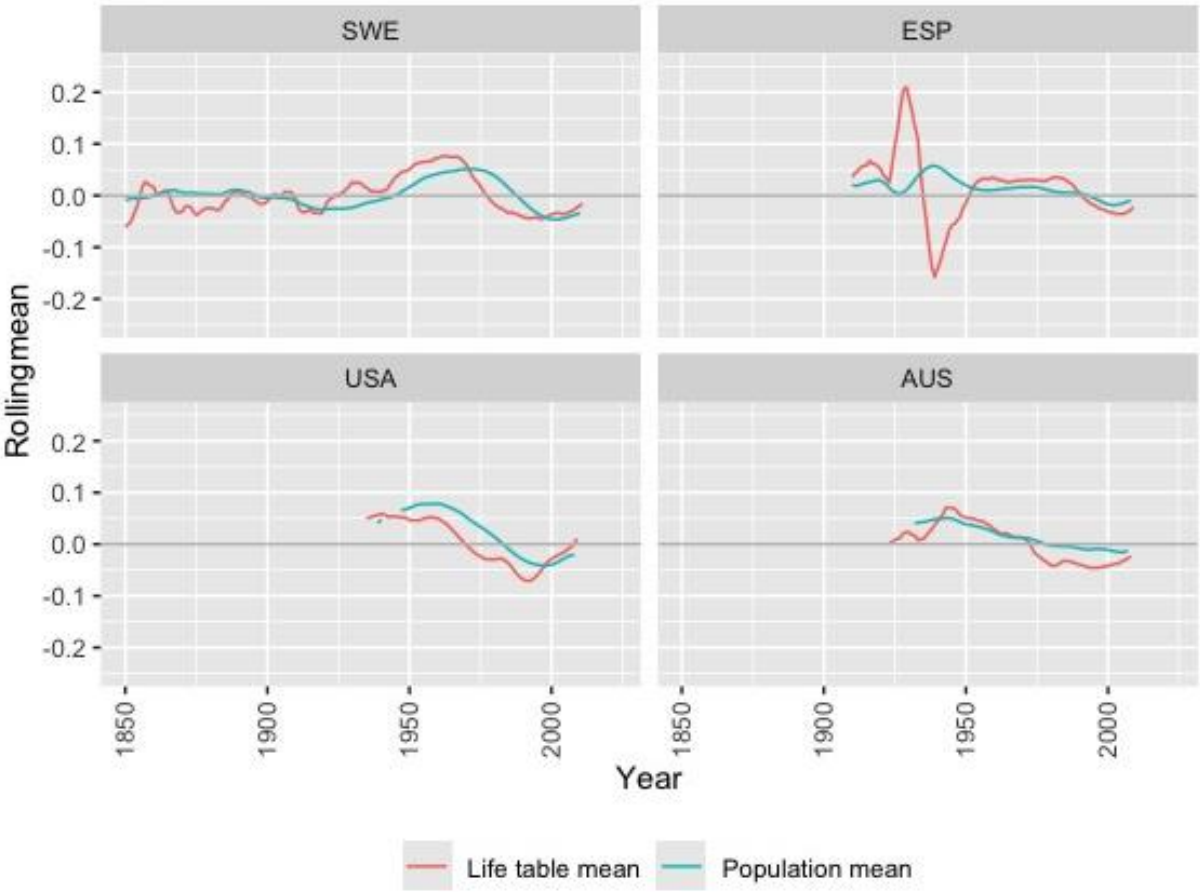
Figure 3: Age-specific contributions to the sex gap in population and life table mean ages by year and country



Source: Human Mortality Database 2024

Figure 4 shows the 10-year derivative with respect to time of the sex gap in the population mean age and life table mean age, that is, the derivative of Figure 2. Positive values indicate a widening in the sex gap over the preceding 10 years while negative values indicate a narrowing of the gap.

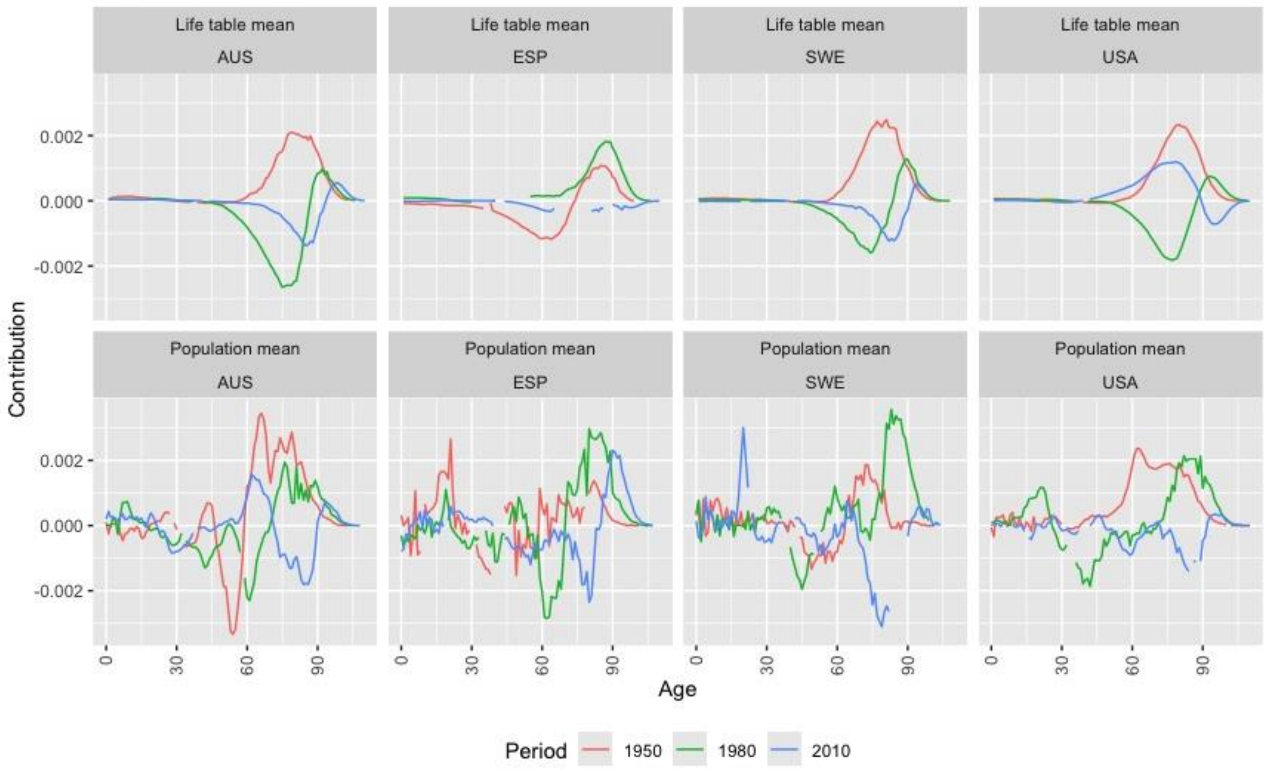
Figure 4: Rolling average of 10-year derivative of sex gap in population and life table mean ages by year and country



Source: Human Mortality Database 2024

Figure 5 shows the age-specific contributions to change over time of the sex gap for both means. It is analogous to Figure 3, but instead of showing age contributions to the sex gap at three discrete time points, 1950, 1980 and 2010, it shows age contributions over time to the change in the sex gap for three time periods, 1950-1960, 1980-1990 and 2010-2020. As Figure 4 shows whether the gap is widening or narrowing over 10-year periods, Figure 5 expands on this by showing age contributions to the widening and narrowing of this gap. As with Figure 3, the patterns between the two means are fairly similar, with significantly more noise in the population mean age than the life table mean age. From 1950-1960, the change over time in the sex gap of the life table mean for Australia, Sweden and the US is at 0 for ages below 60 and positive for ages 60 and above, meaning that ages 60 and above contributed to a widening of the sex gap during this time. Although this positive contribution at ages above 60 can be seen for the population mean age for these countries, there is also volatility at younger ages. As seen in Figure 2 and Figure 4, the period 1980-1990 captures the peak in the sex gap for both means for Australia, Sweden and the US. Figure 5 illuminates an interesting age pattern in this peak: the progression from a widening to a narrowing sex gap does not occur uniformly over all ages. It occurs first at ages 45-85, before including older ages in 2010-2020 for Australia and Sweden. The cases of 1950-1960 in Spain and 2010-2020 in the US also show nonuniformity across ages.

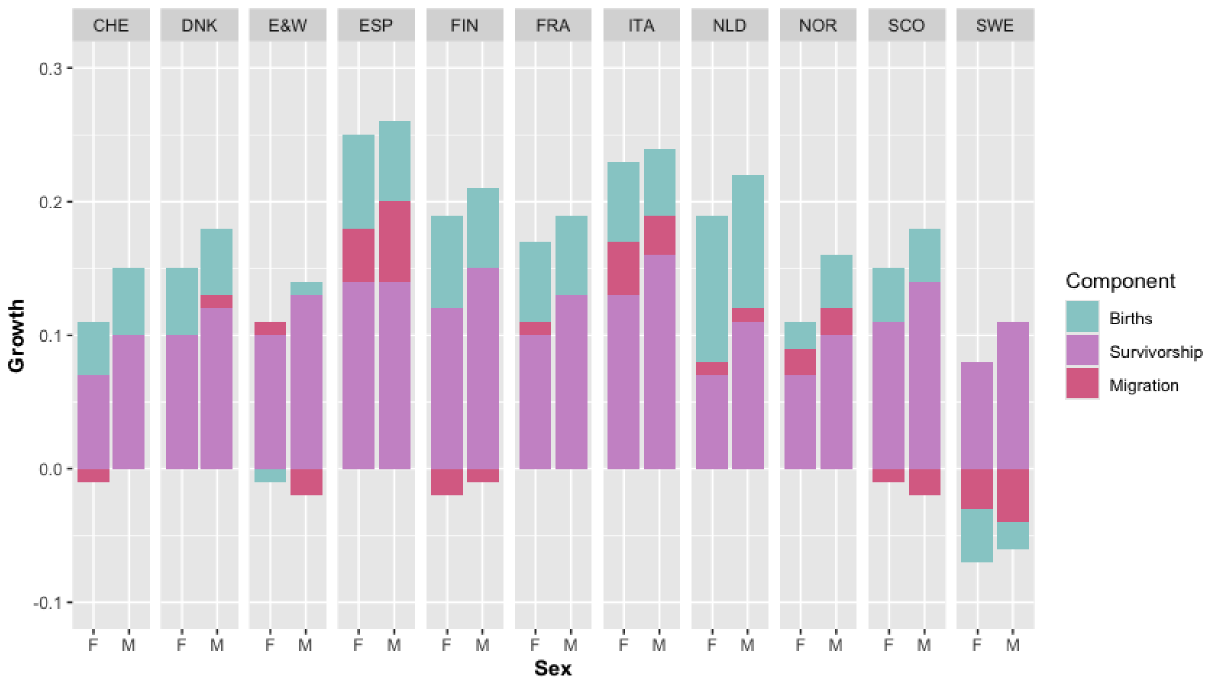
Figure 5: Age-specific contributions to the change in sex gap in population and life table mean ages by 10-year period and country



Source: Human Mortality Database 2024

Figure 6 shows variable-r decomposition of the change in the population mean age from 2010 to 2020 for females and males in various countries (Eq (5)). While the preceding analysis employs a period perspective, Figure 6 takes a cohort perspective, showing how changes in births, survivorship and migration between cohorts throughout the life course contributes to the changing population mean age. Figure 6 shows that changes in survivorship contributes far more than changes in births and migration to the change in the population mean age. Survivorship makes a positive to the change in population mean age for both sexes all countries. Changes in births contributes to ageing in all countries except Sweden, in which changes in births contribute a negative contribution to the change in mean age. The contribution of migration varies between countries. The male population mean age shows greater increase than the female population mean age across countries, meaning that between 2010 and 2020 males aged at a faster pace than females. This is explained by changes in survivorship: historical changes in survivorship are greater for males than females in all countries except Spain. In Spain, the contribution of survivorship to the change in mean age is equivalent.

Figure 6: Variable-r decomposition of change in population mean age by sex, 2010-2020 (Eq (5))



Source: Human Mortality Database 2024

Table 1 continues on from Figure 6, showing variable-r decomposition of the sex gap in the population mean age. Table 1 shows that changes in survivorship explain the majority of the change in the sex gap in the population mean age for all but one country, complementing the findings of the previous figures showing a close relationship between the population mean age and the life table mean age.

Table 1: Variable-r decomposition of change in sex gap in population mean age, 2010-2020

Country	Country code	Sex gap in 2010	Sex gap in 2020	Change in sex gap, 2010-2020	Change in sex gap in births, 2010-2020	Change in sex gap in survivorship, 2010-2020	Change in sex gap in migration, 2010-2020
Italy	ITA	3.00	2.88	-0.13	0.11	**-0.33**	0.09
France	FRATNP	2.94	2.84	-0.1	0.02	**-0.25**	0.12
Finland	FIN	2.89	2.62	-0.26	0.13	**-0.32**	-0.07
Spain	ESP	2.64	2.56	-0.08	0.09	0.04	-0.2
Scotland	GBR_SCO	2.34	2.08	-0.25	-0.04	**-0.31**	0.1
Switzerland	CHE	2.34	2.03	-0.31	-0.06	**-0.24**	-0.01
England & Wales	GBRTENW	2.07	1.89	-0.18	-0.14	**-0.31**	0.26
Sweden	SWE	2.16	1.8	-0.35	-0.15	**-0.31**	0.1
Denmark	DNK	1.98	1.75	-0.22	0.03	**-0.22**	-0.03
The Netherlands	NLD	1.97	1.61	-0.36	0.06	**-0.45**	0.03
Norway	NOR	2.03	1.49	-0.54	-0.16	**-0.34**	-0.05

Source: Human Mortality Database 2024

Conclusions

The title of this study poses the question of whether females and males are ‘growing old together’. This study shows that, for the countries considered, the mean age in the population has increased steadily for males and females, but the female mean age is consistently ahead of the male mean age. The sex gap in the population mean age closely follows the sex gap in the life table mean age, a measure of the age distribution of mortality. Looking at age-specific contributions to these means, it is changes above the age of 60 that explain not only the sex gap at several points in time but also the widening and narrowing of these sex gaps over times. This study contributes to the literature by showing that the sex gap in population ageing can be accounted for by sex differences in mortality at older ages.

References

Horiuchi, S., & Preston, S. H. (1988). Age specifc growth rates: The legacy of past population dynamics. *Demography, 25,* 429–441.

Human Mortality Database. (2024). Rostock, Germany: Max Planck Institute for Demographic Research; Berkeley, CA, USA: University of California, Berkeley; Paris, France: French Institute for Demographic Studies. Available from www.mortality.org

Jdanov, D. A. et al. (2017). Decomposing current mortality differences into initial differences and differences in trends: The contour decomposition method. *Demography, 54,* 1579–1602.

Preston, S. H., Himes, C., & Eggers, M. (1989). Demographic conditions responsible for population aging. *Demography, 26,* 691–704.

Robine, J. & Cubaynes, S. (2017). Worldwide demography of centenarians. *Mechanisms of Ageing and Development*, 165(B), 59-67. https://doi.org/10.1016/j.mad.2017.03.004.

United Nations Department of Economic and Social Affairs, Population Division (2022). *World Population Prospects 2022: Summary of Results*. UN DESA/POP/2022/TR/NO. 3.

Vaupel, J. W., & Canudas-Romo, V. (2002). Decomposing demographic change into direct vs. compositional components. *Demographic Research, 7,* 1–14. https://doi.org/10.4054/DemRes.2002.7.1