The “4-2-1” Family Structure: Simulation Based on the Family Multi-State Life Table

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Introduction

4-2-1 family refers to a family structure consisting of 4 grandparents, 2 parents and 1 child, and both parents are only children, so is the child. The definition of generalized “4-2-1” family structure takes mortality into account, where there may be 0-4 grandparents survived, 0-2 parents survived, 0-1 child survived, that is, it also includes families evolved from the “4-2-1” structure, such as 4-1-1, 2-2-1 family structure and so on. Besides, with the decline in fertility, more and more only children reach the marriage age, “4-2-1” families gradually increase. It is predicted that the number of “4-2-1” family will reach 8.01million in 2025 and 13.47 million in 2035.

The 4-2-1 family structure will bring many consequences, people primarily focus on the pension problem. Many people worry that the “4-2-1” family structure will increase the burden of family elderly care. The parents need to support the four grandparents and one child, or even one child need to support six older adults. However, few research analyze the “4-2-1” family structure specifically.

On the one hand, it is a significant challenge to estimate the mortality issue; on the other hand, the survival situation of family members is complicated. There are possible survival scenarios for grandparents, which results in many obstacles to investigation. Some scholars predict the number of “4-2-1” families based on population microsimulation methods, while others estimated the survival probability and coexistence durations for the narrow definition of “4-2-1” families based on census data, but there are few studies on the evolution of “4-2-1” family structure.

With the decrease of infant mortality, the increase of life expectancy and the postponement of the age of marriage and childbirth, the evolution of the “4-2-1” family structure has also changed. This paper uses life table data and probability method to construct the family multi-state life table, and analyzes the survival probability and existence durations of 30 family structures evolved from the 4-2-1 family. This paper aims to provide a comprehensive analysis for 4-2-1 family and its evolution in China by simulating the development of the 4-2-1 family structure. In order to offer a

foundation to analyze issues related to 4-2-1 family.

Data

The data used in this paper includes life table data and age data. Life table data is based on age-specific mortality data in the national population census in 2020, which is adjusted with Bayesian regression model suitable for incomplete death registration. Two gender-specific life tables were constructed in this study.

The ages at childbirth are based on the ages at first marriage in 1990 and 2020 . According to the population census in 2020, the average age of first marriage is 29.38 for men and 27.95 for women. It is assumed that the age at first marriage for males and females was 29 and 27 in 2020, and they become parents one year after marriage. According to the population census in1990, the average age of first marriage for men and women was 23.59 and 22.15 respectively. It is assumed that the age at which men and women became parents in 1990 was 25 and 23 respectively. That is, when the child is born, the paternal grandfather is 55, the paternal grandmother is 53, the maternal grandfather is 53 and the maternal grandmother is 51, and the parents are 30 years old and 28 years old respectively. The survival probability and existence durations of 4-2-1 family structure are analyzed based on it.

Method

Definition

In the life table, l_x represents the population of x years old, ${}_nP_x$ is the probability of surviving from age x to $x+n$ years old, it can be expressed as:

$$ {}_nP_x = \frac{l_{x+n}}{l_x} \tag{1}$$

After the birth of the child, 4-2-1 family comes into effect. We take the age of child as the standard and defines the initial age of the child as 0 in this paper. The age difference between the child and the father, mother, paternal grandfather, paternal grandmother, maternal grandfather and maternal grandmother are labeled f, m, ff, fm, mf, and mm, respectively. The events in probability are used to define individual survival as shown in the Table 1.

Table 1. Key to event labels

A_1	The child survival event
A_2	The father survival event
A_3	The mother survival event
A_4	The paternal grandfather survival event
A_5	The paternal grandmother survival event
A_6	The maternal grandfather survival event
A_7	The maternal grandmother survival event
B	All four grandparents coexistence event
C	At least three grandparents survival event
D	At least two grandparents survival event
E	At least one grandparent survival event
F	No grandparent survival event
G	All two parents survival event
H	At least one parent survival event
I	No parent survival event

Survival probability

The death of family members is independent, the events are independent. The survival probability of family members from λ years old to $\lambda+n$ years old can be shown as:

$$P(A_i)=\frac{l_{\lambda+n}}{l_\lambda},\ \ i=1,\cdots,7,\ \ \lambda=\{f,m,ff,fm,mf,mm\} \tag{2}$$

The above formulas refer to data from the male and female life tables, respectively, based on the gender of the event object.

The survival probability of all four grandparents is:

$$P(B)=P(A_4\bigcap A_5\bigcap A_6\bigcap A_7)=\prod_{i=4}^{7}P(A_i) \tag{3}$$

The survival probability of at least three grandparents is

$$\begin{aligned}P(C)&=P(A_4A_5A_6\bigcup A_4A_5A_7\bigcup A_4A_6A_7\bigcup A_5A_6A_7)\\&=\sum_{4\le i<j<k\le 7}P(A_i)\times P(A_j)\times P(A_k)-3\prod_{i=4}^{7}P(A_i)\end{aligned} \tag{4}$$

The survival probability of three grandparents is $P(C)-P(B)$.

The survival probability of at least two grandparents is:

$$\begin{aligned}P(D)&=P(A_4A_5\bigcup A_4A_6\bigcup A_4A_7\bigcup A_5A_6\bigcup A_5A_7\bigcup A_6A_7)\\&=\sum_{4\le i<j\le 7}P(A_i)\times P(A_j)-2\sum_{4\le i<j\le 7}P(A_i)\times P(A_j)\times P(A_k)+3\prod_{i=4}^{7}P(A_i)\end{aligned} \tag{5}$$

The survival probability that only two of the four grandparents is $P(D)-P(C)$.

The survival probability that at least one of the four grandfathers is:

$$P(E)=P(A_4\bigcup A_5\bigcup A_6\bigcup A_7)$$
$$=\sum_{i=4}^{7}P(A_i)-\sum_{4\le i<j\le 7}P(A_i)\times P(A_j)+\sum_{4\le i<j\le 7}P(A_i)\times P(A_j)\times P(A_k)-\prod_{i=4}^{7}P(A_i) \quad (6)$$

The survival probability that only one of the four grandfathers is $P(E)-P(D)$.

The probability that none of the four grandfathers survives is

$$P(F)=1-P(E) \quad (7)$$

The survival probability of both parents is:

$$P(G)=P(A_2)\times P(A_3) \quad (8)$$

The survival probability that at least one parent is:

$$P(H)=P(A_2)+P(A_3)-P(G) \quad (9)$$

The survival probability of only one parent is $P(H)-P(G)$.

The probability of no parents survive is:

$$P(I)=1-P(H) \quad (10)$$

After calculating the probability of the above events, the survival probabilities of 30 family structures, such as 4-2-1 family and 3-2-1 family, are calculated.

The survival probability of 4-2-1family structure is

$$P_x^{421}=P(B)\times P(G)\times P(A_1) \quad (11)$$

4-0-1 family structure refers to four grandparents survive, two parents die, and the child survive. The survival probability of 4-0-1 family structure is

$$P_x^{401}=P(B)\times P(I)\times P(A_1) \quad (12)$$

3-2-1 family structure refers to three grandparents, two parents and the child survive. The survival probability of 3-2-1 family structure is:

$$P_x^{321}=(P(C)-P(B))\times P(G)\times P(A_1) \quad (13)$$

Existence Duration

L_x stands for the duration of every family structure when the child is from 0 to N years old. So according to the life table calculation method, based on the age of the child, in each year from 0 to N, the duration of each family structure is:

$$L_x = \frac{P_x + P_{x+1}}{2}, \quad x = 1, 2, \cdots, N \tag{14}$$

Corresponding to Equation (11), the duration of 4-2-1 family in each year from 0 to N is:

$$L_x^{421} = \frac{P_x^{421} + P_{x+1}^{421}}{2}, \quad x = 1, 2, \cdots, N \tag{15}$$

By adding up the survival years of each family structure at the child's different ages, we can obtain the number of years each family structure will exist after the birth of the child.

$$e_0 = \sum_{i=0}^{N+1} L_i = \sum_{i=0}^{N} \frac{P_i + P_{i+1}}{2} + L_{N+1} \tag{16}$$

After the birth of the child, the age-specific expectancy of the life of each family structure can be expressed as:

$$e_x = \sum_{i=x}^{100} L_i \tag{17}$$

After the birth of the child, the age-specific years have existed of each family structure can be expressed as

$$T_x = e_0 - e_x \tag{18}$$

Results

Age-specific survival probability of "4-2-1" family structure

Figure 1 shows the age-specific survival probability of the "4-2-1" family structure. After the birth of the child, the probability of the 4-2-1 family structure decreases sharply with the increase of the age of four grandparents and two parents, and the 4-2-1 family gradually develops into 3-2-1, 2-2-1, 0-0-1 and other families, until all seven people in the 4-2-1 family die. After the child is 9 years old, the four grandparents are all aged over 60 years old. The survival probability of 4-2-1 family structure decreases to 0.79 when the child were 9 years old. The survival probabilities of family structure evolved from 4-2-1 family structure first increases and then decreases by the age of child . The survival probability of 3-2-1 family structure peaks at 0.40 when the child is 22 years old. The survival probability of 2-2-1 family structure peaks at 0.33 when the child is 30 years old. The survival probability of 1-2-1 family structure peaks at 0.33 when the child is 36 years old. The survival probability of 1-1-1 family structure peaks at 0.10 when the child is 39 years old, and it peaks at 0.45 for 0-2-1 family structure when the child is 44 years old. After the child is 50 years old, none of four grandparents survives, and the survival probability of 0-1-1 family structure peaks at 0.49 when the

child is 53 years old. Finally, the survival probability of 0-0-1 family structure peaks at 0.72 when the child is 71 years old.

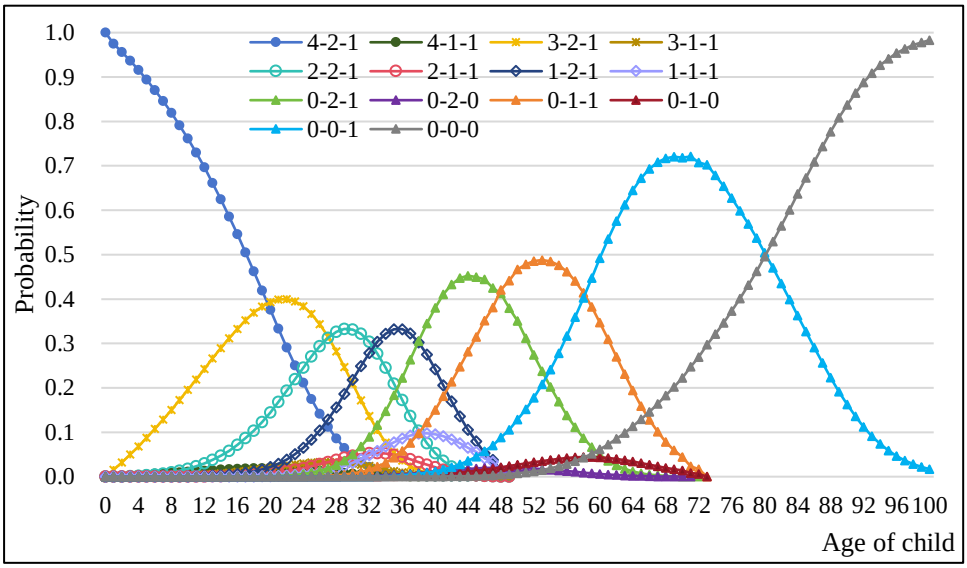

Figure 1 Age-specific survival probability curve 4-2-1 family structure

Notice: Figure 1 only displays family structures that the peak value of age-specific survival probability greater than 0.01. The other 16 family structures are not shown, as their age-specific survival probability consistently remains below 0.01.

The existence duration of "4-2-1" family structure

Figure 2 shows the existence duration of the "4-2-1" family structure. The years of coexistence during which all four grandparents are living is 17.24 years. During this period, when the child is from 0 to 9 years old, all four grandparents are under 60 years, while when the child is aged from 10 to 17, the grandparents pass into "old age" (over 60 years old) but still remain in a younger old-age period. When the child reaches 17, one of the grandparents will die, and for the 9 years, when the child is aged from the 17 to 26, only three grandparents survive, whose ages are between 68 and 81. when the child is 26, only two grandparents remain living, and the existence duration of which is 6.38 years. when the child is aged from the 32 to 38, only one of the grandparents will still be living.

Figure 2. Survival years of grandparents in 4-2-1 family structure

Figure 3 shows the years of coexistence of three generations in the 4-2-1 family. The survival probability of the family structure with both parents and children surviving is higher among all family structures evolved from 4-2-1 family. Under the condition that both parents and one child survive, the years of coexistence during which all four grandparents are living is 16.67 years, the years of coexistence during which at least one person is living is 35.19 years, and the existence duration of only one of grandparents is living is 4.91 years.

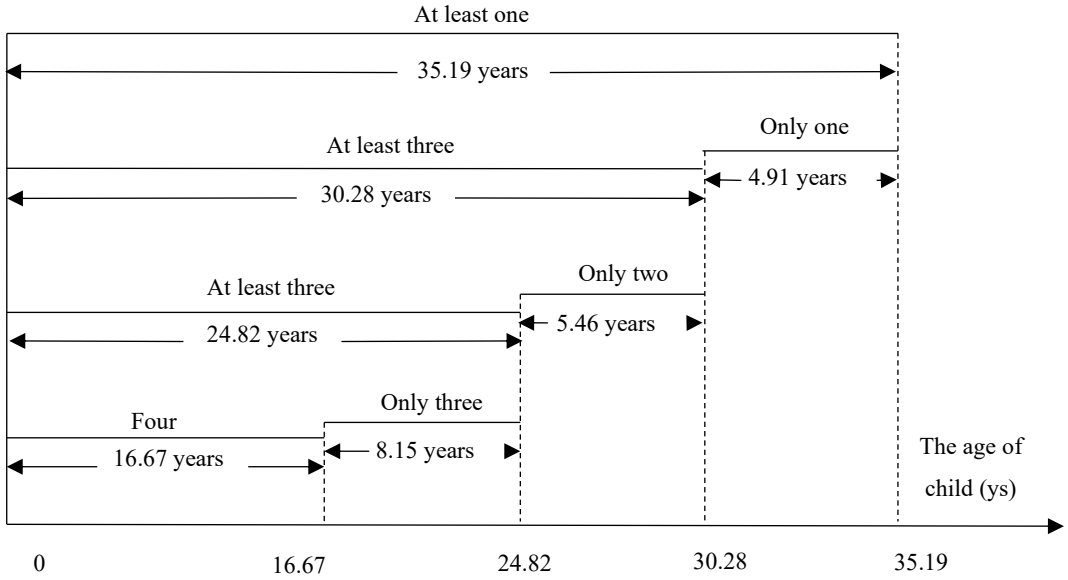

Figure 3. Survival years of grandparents in 4-2-1 family structure, conditional on the survival of two parents and one child

Figure 4 displays the age-specific expectancy of the life of "4-2-1" family structure. At the birth of the child, the expectancy of the life of the 4-2-1 family structure is 16.67 years, and it declines sharply when the child is aged from 0 to 45. The expectancy of the life of 3-2-1 family structure is 8.15 years. The expectancy of the life of 2-2-1 family structure is 5.46 years, and decreases significantly since the child is 18. The expectancy of the life of the 1-2-1 family structure is 4.91 years, and decreases significantly since the child is 22. The expectancy of the life of 1-1-1 family structure is 1.35 years, and it decreases significantly since the child is 35. As all the grandparents die, the expectancy of the life of 0-2-1 family structure is 8.25 years, and it decreases significantly since the child is 32. The expectancy of the life of 0-1-1 family structure is 10.02 years, and there is no obvious change before the child reaches 31. After grandparents and parents all die, the expectancy of the life of 0-0-1 family structure is 20.34 years, and there is no significant change before the child reaches 48 years old. Other family structures that evolved from the 4-2-1 family has very low life expectancy.

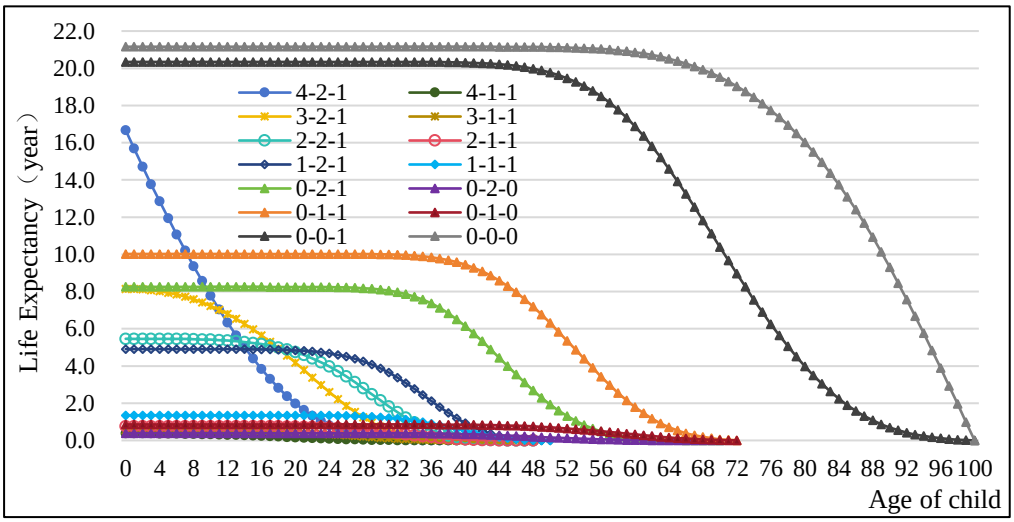

Figure 4. The age-specific expectancy of the life of "4-2-1" family structure

Notice: The other 16 family structures are not shown, as their age-specific survival probability consistently remains below 0.01.

Figure 5 shows the age-specific years that each "4-2-1" family structure has survived. The "4-2-1" family is formed after the birth of child, and the year that 4-2-1 family structure has survived is 16.60 years when the child is 31, and there is barely change after that. The 3-2-1 family structure has survived for 1.12 years when the child is 11 and it increases to 8.10 years when the child is 37, after which there is almost no change. The 2-2-1 family structure has survived 1.05 years when the child is 22, and it rises to 5.42 years when the child is 42, and almost no change before the child reaches 48. The 1-2-1 family structure has survived for 1.03 years when the child is 30, and the duration peaks at 4.91 years when the child is 50. The 1-1-1 family structure has

survived for more than 1 year when the child is 42, and the family structure will not exist at the child's age of 50. As all four grandparents die, the durations that 0-2-1 family structure has survived are 1.13 years at the child's age of 37, and which peaks at 8.21 years when the child is 63, after which there is almost no change. The 0-1-1 family structure has survived 1.16 years when the child is 43. The 0-0-1 family structure has survived more than one year when the child is 73.

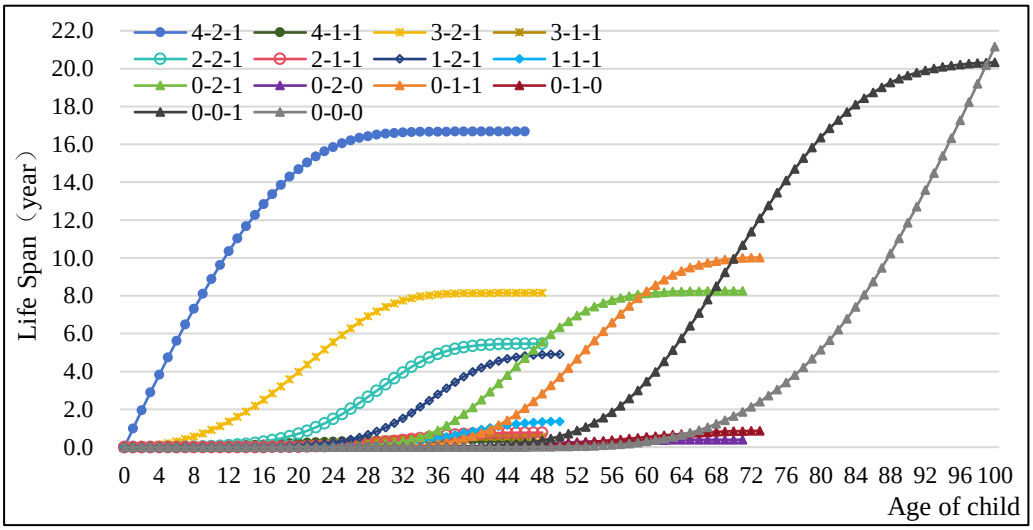

Figure 5. The years that each "4-2-1" family structure has survived

Notice: The other 16 family structures are not shown, as their age-specific survival probability remains below 0.01.

Conclusion

Based on the life table data and probability method, this paper discusses the survival probability and existence durations of "4-2-1" family, the results show:

Firstly, after the birth of the child, the four grandparents, two parents and one child coexistence for 16 years, when the maximum age of the four grandparents is no more than 71 years old. Therefore, the burden of supporting four elderly people in the "4-2-1" family is not prominent.

Secondly, with the increase of the age of child, the survival probability of 4-2-1 family structure decreases sharply. When the child reaches 9 years old, the grandparents are all aged over 60, the probability of 4-2-1 family structure decreases to 0.79. The 4-2-1 family gradually develops until all seven members die. The survival probabilities for other family structure evolved from "4-2-1" family first rise and then fall by the age of child, and the survival probabilities are higher for family structures that both parents and children survive.

Finally, after the only child are 35 years old, and the parents are over 65 years old. The expectancy of the life of the 0-2-1 family structure is 7.57 years when the child is 35. In the "4-2-1" family, it barely exists that the parents support four older adults and the child support five or six older adults.