On the use of Kannisto model for mortality trajectory modelling at very old ages

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Abstract

The Kannisto model is one of the most widely used parametric models for old-age mortality estimation. The model finds extensive use in life tables calculations, such as in the Human Mortality Database (HMD, 2023) for old-age mortality smoothing, and in pension and annuity policy planning. However, recent concerns have arisen regarding the accuracy of this model for modeling mortality patterns at advanced ages (Feehan, 2018). We explore the question further using thorough age-validated mortality data for France, Belgium and Quebec (Canada), where a large number of extinct birth cohorts were followed up to age 115. Our comparison of the Kannisto model's performance with other frequently used mathematical mortality models showed a systematic underestimation of death rates for the Kannisto model, which becomes apparent as early as circa age 100 in all populations studied, for both males and females. This systematic underestimation in mortality would translate in a systematic overestimation of life expectancy that becomes noticeable after age 100. Caution is thereby advised when using the Kannisto model to estimate mortality at very old ages.

Introduction

To model mortality data, parametric models have long been the standard approach, with broad usage. Among these, the Kannisto model is one of the most popular models to date. Introduced to the scientific community for the first time in June 1992 by Väinö Kannisto at a workshop on old-age mortality at the University of Southern Denmark in Odense (Kannisto, 1992), the simplified logistic model, which later became known as the Kannisto model, has long stood as perhaps the most prominent model for estimating the age trajectory of mortality at very old ages. Capable of capturing the decelerating trend in the force of mortality,— a characteristic for which the family of logistic models is most renowned,— the fact that the Kannisto model does so with only two parameters instead of three earned it the endorsement of researchers in the field of mortality studies. This model found extensive application in several major practical frameworks, including in the development of life tables in the Human Mortality Database (HMD, 2023), and also in that of pensioners and annuitant groups both in public and private sectors. However, more recently, evidence has emerged highlighting the need for caution when using the Kannisto model (Feehan, 2018). With continuous increase of life expectancy, especially in low-mortality countries, it is only sensible to expect growing number of survivors to these oldest ages (Global Burden Disease Forecast, 2024). Modeling accurately the force of mortality at these ages using appropriate models therefore becomes ever more an urgent necessity than pure scientific curiosity.

In this paper, we investigate the performance of Kannisto model further by comparing it to that of frequently used models for depicting the trajectory of mortality at the most advanced ages. We use a set of mortality data for three populations where all birth cohorts were followed until the age of 115 or higher, rather than being limited to much *younger* old ages that previous study done by Feehan (2018) was constrained to due to the availability of data. Moreover, the data we use is subject to a thoroughly age validation protocol that meets the highest criteria from International Database on Longevity (2023) as described in the next section.

Data

The data that we use in this study come from three populations, namely France, Belgium, and Quebec (Canada). The historical depth of mortality data collected in these populations allow us to study extinct birth cohorts, and also to apply a strict age validation process adapted to the specific characteristics of each population. This protocol guarantees two key features making these mortality data at oldest ages highly reliable: the data on deaths are exhaustive and as accurate as can be according to the information available in each country. The data and their sources are presented in the table 1. We compute aggregated number of deaths by country/region, sex, age, year of birth, and year of death. The exhaustive lists of individual deaths from which our aggregation death counts come from include information about the sex, date of birth and date of death of all deceased individuals. We model mortality starting from age 90, and while these data are deemed confidential and were obtained under strict protocol, an anonymized version of these data for ages 105 and above is publicly available in the International Database on Longevity (IDL, 2023), free of charge after registration.

Table 1: Description of the mortality data for French, Belgian and Quebec populations

Description	**France**		**Belgium**	**Quebec**
Source of data	Institut national de la statistique et des études économiques		Registre national	Institut de la statistique du Québec
Type of data	Aggregated	Individual	Individual	Individual
Total death count (all sexes and ages combined)	1,028,208		140,990	31,794
Completed ages at death	90–104	105+	90+	90+
Birth cohorts	1883–1901		1891–1904	1880–1896
Years of death	1973–2016		1981–2015	1970–2009

Methods

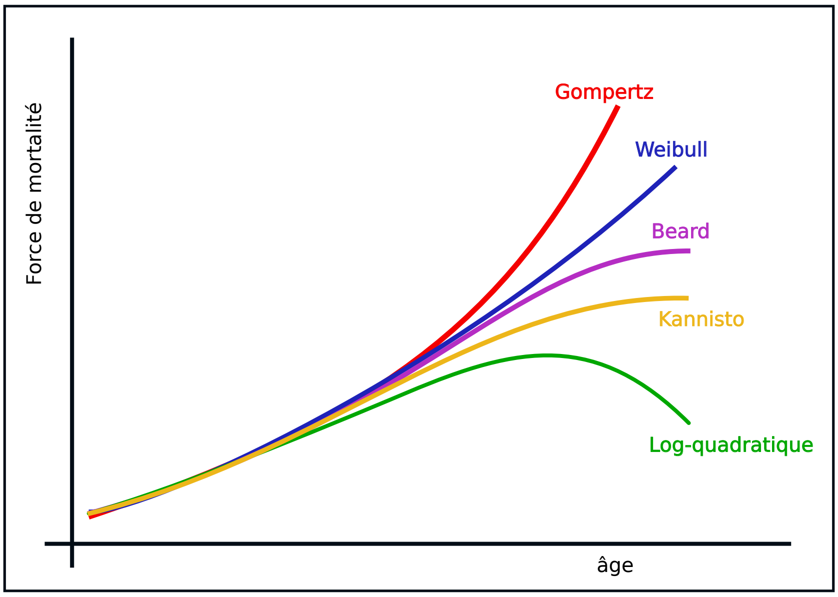
Selected mortality models

The Kannisto model belongs to the family of logistic models that were proposed firstly by Perks in 1932 and further developed and put into practice by Beard (1952, 1971). Similarly to all functions belonging to this family, the Kannisto model is characterized by two main features. The first is the existence of an inflection point, after which the rate of increase of mortality could change, by starting to slow down. This phenomenon is known as old-age mortality deceleration. The second inherent mathematical property of the Kannisto model is the existence of a horizontal asymptote, often interpreted in the literature as the mortality plateau. As opposed to other models belonging to the logistic family that have an asymptote at a level that varies depending on data, the Kannisto model has a fixed asymptote at unity. The mathematical expression describing the Kannisto model is given by

$$\mu(x) = \frac{a\, e^{b\,x}}{1 + a\, e^{b\,x}},$$

where $\mu(x)$ is the force of mortality expressed in function of age x. In this paper, we compare the performance of the Kannisto model by comparing it to several competing models which are the other most widely used ones for modelling mortality at very old ages : the Gompertz model (Gompertz, 1825), the Weibull model (Weibull, 1951), the log-quadratic model (Coale and Kisker, 1990), and another model that belongs to the logistic family — the Beard model (Beard, 1952). These models describe the most frequent scenarios for the trajectory of mortality at the highest ages which are, respectively, an exponential growth of mortality, a linear growth, a potential decrease, and a deceleration of mortality with an horizontal asymptote to be estimated from the data. Figure 1 gives an overview of these various scenarios.

Figure 1: Scenarios of mortality trajectory at highest ages and their corresponding possible mortality models



The mathematical expressions for fitting the four models to be compared with the Kannisto model, based on observed data, are the following:

$$\text{Gompertz model: } \mu(x) = a\, e^{b\,x},$$

$$\text{Weibull model: } \mu(x) = ax^b,$$

$$\text{log-quadratic model: } \mu(x) = \exp(ax^2 + bx + c),$$

$$\text{Beard model: } \mu(x) = \frac{a \exp(bx)}{1 + \delta \exp(bx)},$$

where the horizontal asymptote δ/a in the Beard model is to estimated from the data.

Fitting the models to empirical mortality data

We assume that the death counts are realizations of a Poisson distribution, namely

$$D_x \sim \mathcal{P}(\mu_x(\boldsymbol{\theta})\, E_x),$$

where D_x is the number of deaths observed at each age x, μ_x is the force of mortality at age x, and E_x is the exposure-to-risk, also at age x. We fitted the selected models to our age verified empirical data using the method of maximum of likelihood, where the likelihood function to be maximized is given by

$$\ell(\boldsymbol{\theta}|D_x, E_x) \propto \sum_{x=x_0}^{\omega} \left(D_x \ln \mu_x(\boldsymbol{\theta}) - \mu_x(\boldsymbol{\theta})\, E_x\right).$$

It should be mentioned that for no reason other than plain numerical challenges during the fitting process, the log-quadratic model was fitted to data using a generalized linear model (GLM) approach, while other models were fitted through the method of maximum likelihood. These two routines provided similar estimations for our model parameters and similar estimated forces of mortality at each age. As a precaution, we used deviance residuals for computing the AIC by applying the following formula

$$\text{AIC} = \text{Dev} + 2\, m,$$

where m is the number of parameters included in the model, and Dev is the deviance residuals given by

$$\text{Dev} = 2const - 2 \ln \mathcal{L}(\text{proposed model}|\text{data}),$$

where $\ln \mathcal{L}(\text{proposed model}|\text{data})$ is the value of the log-likelihood of the given model estimated for each data set. With this alternative approach, the rule is still such that the smaller AIC yields the better performing model.

Results

Visual inspection

Figures 2, 3, 4, and 5 show the sex-specific observed death rates and the fitted curves according to the selected mortality models, respectively in French, Belgian, and Quebec populations, in addition to all three populations pooled together. In all female data sets, the Kannisto model systematically provides the lowest estimates of mortality above age 100 compared to other models. That remains true even when it is compared to the Beard model that also has the ability to accommodate for a mortality plateau at very old ages and the log-quadratic model that allows for an even wider amount of flexibility. In male populations, not only does the Kannisto model result in the lowest mortality estimates at the highest ages, but it becomes very apparent that it no longer fits observed male death rates well at the highest ages. For both sexes, the Kannisto curves seem to bend prematurely and are positioned far lower compared to its logistic model counterpart (i.e. the Beard model) and the log-quadratic model.

Interestingly, it is also noticeable that in most Figures 2–5, the Kannisto-fitted curves lie well above the dotted horizontal line, placed at a level of 0.7 for the force of mortality, which is often cited in the literature as the level of the human mortality plateau. Further work,— beyond the scope of the current paper,– is needed to verify this observation. However, this could suggests that if the plateau of mortality exists at extreme ages, its level must be higher than 0.7.

Figure 2: Fitted force of mortality (with 95% CIs), French female (left) and male (right) populations, birth cohorts 1883–1901

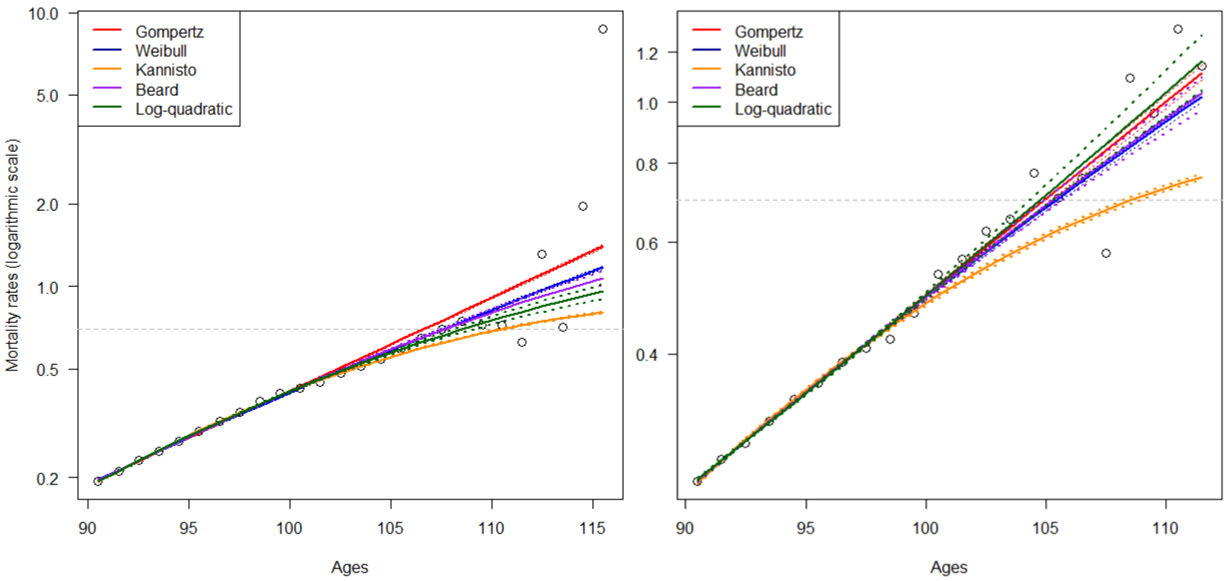


Figure 3: Fitted force of mortality (with 95% CIs), Belgian female (left) and male (right) populations, birth cohorts 1891–1904

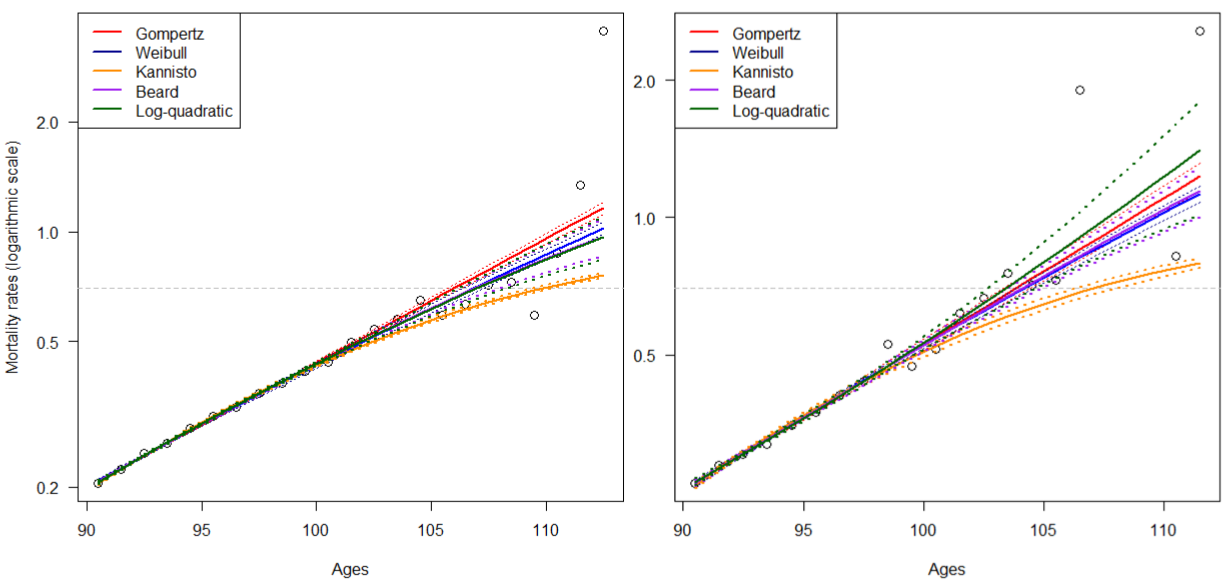


Figure 4: Fitted force of mortality (with 95% CIs), Quebec female (left) and male (right) populations, birth cohorts 1880-1896

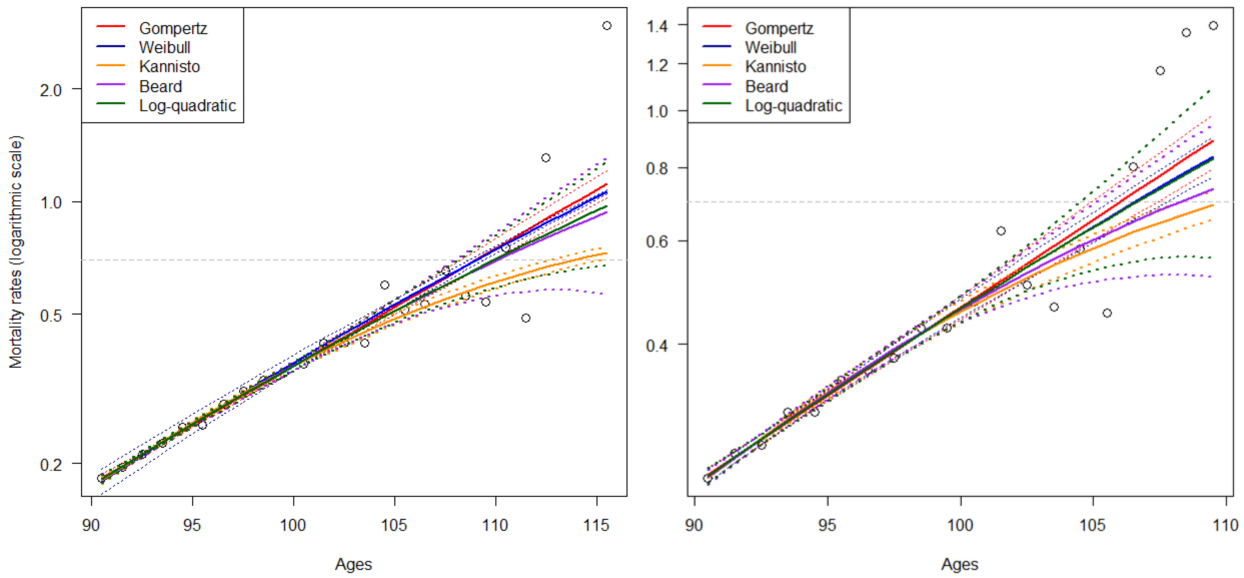
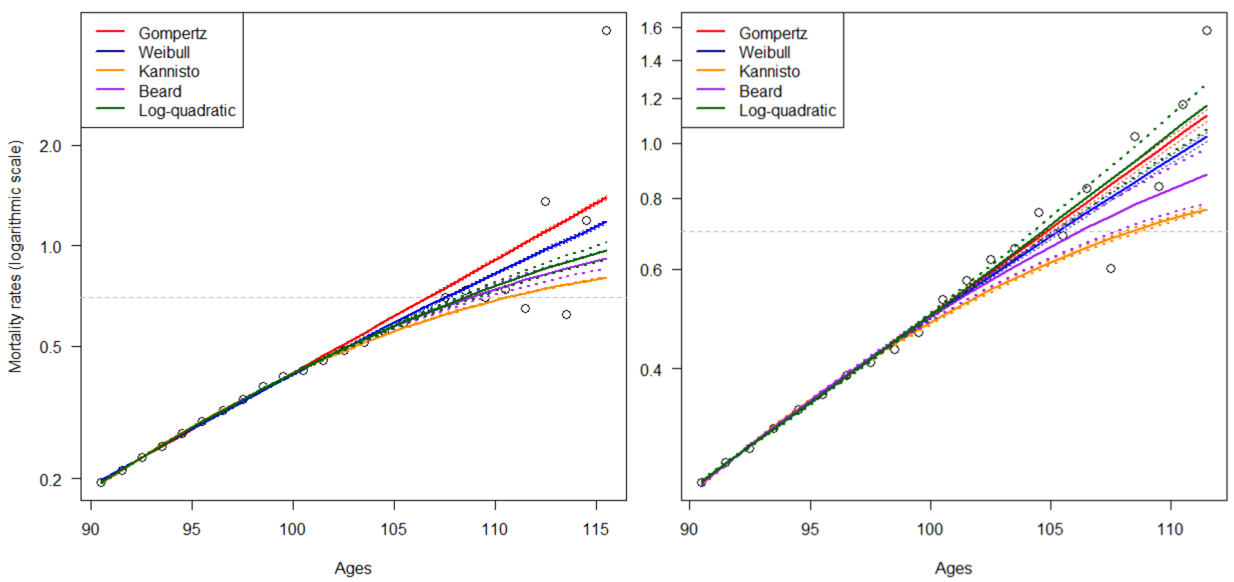


Figure 5: Fitted force of mortality (with 95% CIs), pooled female (left) and male (right) populations, birth cohorts 1880-1904



Analytical results

The tables 2 and 3 give the AIC values for each model in each data set, respectively for female and for male populations. For both sexes, even if the optimal model changes according to the population, the Kannisto model does not qualify as the optimal model in any of our data sets. For female populations, when the number of observations is considerably reduced, such as in the cases of Quebec and to a lesser extent Belgian populations, the Kannisto model performs the worst. Among males, the Kannisto model also exhibits the poorest performance, except for the Quebec population. There is more than just the number of observations to consider when it comes to the performance of a regression model, but the fact that the Kannisto model does not perform particularly well in any of our data sets, which are made of the most recent and high quality mortality data currently available. This could decidedly give way for concern, especially given that the Kannisto model has been serving as a widely used standard to estimate the age trajectory of mortality at very old ages.

There are several reasons for the poor performance of the Kannisto model on the data included in this study. Among them, fixing an horizontal asymptote at unity for the force of mortality might be still too low for the current mortality trends at the highest ages. And even if mortality deceleration at the population scale is the *true* underlying pattern at very old ages, for the time being, human mortality seems to continue increasing at a greater pace than what is implied to reach a plateau at unity. This observation could be counted as one additional material to re-consider whether the mortality plateau, assuming it exists, would be only around 0.7 as it has been suggested by several earlier studies (Gampe, 2010, 2021; Rau et al., 2017).

Table 2: Mortality model rankings according to AIC, France, Belgium, Quebec and pooled female populations

Mortality model	**France (c.1883-1901)**		**Belgium (c.1891-1904)**		**Québec (c.1880-1896)**		**Pooled (c.1880-1904)**	
	ΔAIC	Rank	ΔAIC	Rank	ΔAIC	Rank	ΔAIC	Rank
Gompertz	158.53	5	6.09	4	0.00	1	172.03	4
Weibull	97.33	4	1.99	3	2.64	4	92.16	5
Kannisto	15.58	2	9.35	5	2.81	5	25.73	3
Beard	37.31	3	1.07	2	1.59	2	0.00	1
Log-quadratic	0.00	1	0.00	1	1.63	3	2.29	2

Table 3: Mortality model rankings according to AIC, France, Belgium, Quebec and pooled male populations

Mortality model	**France (c.1883-1901)**		**Belgium (c.1891-1904)**		**Québec (c.1880-1896)**		**Pooled (c.1880-1904)**	
	ΔAIC	Rank	ΔAIC	Rank	ΔAIC	Rank	ΔAIC	Rank
Gompertz	0.00	1	0.01	2	0.23	2	0.00	1
Weibull	6.49	4	2.69	3	0.03	1	7.67	3
Kannisto	63.04	5	17.52	5	1.16	3	75.24	5
Beard	5.57	3	3.33	4	2.56	5	34.91	4
Log-quadratic	1.32	2	0.00	1	2.01	4	1.27	2

Going one step further, the age at which the Kannisto model starts to diverge from its competing mortality models could be an useful information, namely for practitioners' involved in the decision making of which mortality model to use for modeling mortality at the highest ages, given their study population and their question(s) at hand. One straightforward way to do so is to estimate the age at which the mortality estimates from the Kannisto model differ statistically from those estimated by other models. Herein, using model-specific 95% confidence intervals, we can estimate the age at which the upper band of the confidence interval band of the alternative model no longer includes the Kannisto fitted values. Tables 4 and 5 report the various ages of separation for the Kannisto model with respect to the other selected models, respectively for pooled female and male populations. Compared to the optimal models that were selected earlier by AIC (i.e., Beard model for females and Gompertz model for males), the separation started as early as at age of 101.51 for the first case and 99.33 for the later, and the underestimation seems to become ever more apparent with further advanced ages. With respect to all of the alternative models, the separation ages are between 100–102 years old for females and 99–101 for males, suggesting that caution is advised as early as when ages of centenarians are reached.

Table 4: Comparison between the Kannisto model and selected competing models using ΔAIC, and estimated age of separation, pooled female population, birth cohorts 1880–1904

Model	Competing models	Age of separation
Kannisto	Gompertz	100,23
	Weibull	103,73
	Beard	**101,51**
	Log-quadratic	101,82

Note: The Beard model marked in bold is the optimal model according to AIC for pooled female population (see Table 2).

Table 5: Comparison between the Kannisto model and selected competing models using ΔAIC, and estimated age of separation, pooled male population, birth cohorts 1880–1904

Model	Competing models	Age of separation
Kannisto	**Gompertz**	**99.23**
	Weibull	99.55
	Beard	101.23
	Log-quadratic	99.25

Note: The Gompertz model marked in bold is the optimal model according to AIC for pooled male population (see Table 3).

Tables 6 and 7 report estimated life expectancy at different given ages using each model under study. As what was informed by the ages of separation between Kannisto model and competing models, there is almost no difference between estimated life expectancy from age 0 to 85 between competing models. However, starting around age 100, the gap between estimations starts to show. Kannisto model gives an estimation that is visibly higher than not only models that predict a continuous exponential growth of mortality like Gompertz model or model of weaker deceleration like Weibull model, but it is also higher than model of the same logistic family like Beard model and model that could predict even a diminution of mortality at highest ages like log-quadratic model. Precisely, Kannisto model overestimates the life expectancy at age for females by 0.06 and 0.10 at age 110 (compared to log-quadratic model). For males, this overestimation is of 0.23 at age 100 and 0.38 at age 110 (compared to optimal model: Gompertz model). This overestimation of life expectancy above age 100 is the direct consequence of Kannisto model systematically underestimating mortality at these higher ages.

Table 6: Life expectancy estimated at different ages by all models under study, French females, birth cohorts 1883-1901

	Gompertz	Weibull	**Kannisto**	Beard	Log-quadratic
e0	52.55759	52.5573	**52.55776**	52.5577	52.55769
e65	16.7745	16.77394	**16.77484**	16.77471	16.77471
e85	5.60777	5.60626	**5.60869**	5.60834	5.60834
e100	1.50902	1.59363	**1.72207**	1.66152	1.64854
e110	1.05667	1.17824	**1.43269**	1.33396	1.29834

Table 7: Life expectancy estimated at different ages by all models under study, French males, birth cohorts 1883-1901

	Gompertz	Weibull	**Kannisto**	Beard	Log-quadratic
e0	42.01528	42.0153	**42.0153**	42.01529	42.01529
e65	13.1234	13.12343	**13.12345**	13.12342	13.12341
e85	4.56961	4.56975	**4.56982**	4.5697	4.56967
e100	1.35834	1.40645	**1.58747**	1.35992	1.33689
e110	0.96342	1.04136	**1.34675**	0.96687	0.92852

Conclusion

Based on the most recent thoroughly age-validated data for extinct birth cohorts from France, Belgium and Quebec (Canada), our results indicate that the well known and widely used Kannisto model systematically underestimates mortality at the oldest ages. The higher the ages, the more apparent it becomes that the mortality deceleration resulting from the Kannisto model is greater than suggested by empirical data. The underestimation seems to start as early as age 101 for females and 99 for males. The horizontal asymptote fixed at unity forces the mortality curve to bend prematurely and does not depict the trajectory of mortality at the highest ages as accurately as the Beard model – another model belonging to the logistic family. The use of the Kannisto model for estimating old-age mortality trajectories should therefore be made with great precaution. Other competing models, such as the Beard model or the log-quadratic model, appear as the most appropriate candidates for substitution.

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