Assessing age-specific contributions to life expectancy gaps: the Proportional Difference Comparison Method

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Abstract

A central goal of demographic analysis is to better understand life expectancy gaps between populations, such as by sex, socioeconomic status, or race/ethnicity. Because life expectancy increases more if the same proportional improvement occurs at younger vs. older ages, all else being equal, existing demographic decomposition methods may lead researchers to respectively overstate and understate the importance of mortality differences at younger and older ages. Here, we propose to compare (a) observed age-specific contributions to a given life expectancy gap with (b) hypothetical age-specific contributions that would be observed if the mortality rates between two populations differed by the same relative factor at all ages, but the life expectancy gap remained fixed. We apply the *Proportional Difference Comparison Method* (PDCM) to the Black–white life expectancy gap in the United States, showing that observed contributions of infant and young adult mortality exceed expected contributions to the gap by the same factor. In contrast, contributions of older ages are smaller than expected under the proportional scenario. We argue that PDCM can be used in comparative mortality analysis to better understand the biological and social factors shaping age-specific contributions to life expectancy gaps between populations.

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1 Introduction

Life expectancy differences are a key subject of mortality research. Whether differences are between countries or at the subnational level—such as by sex, socioeconomic status, or race/ethnicity—life expectancy gaps are thought to reflect underlying biological differences or differences in access to health-promoting resources and behaviors [Mackenbach, 2012], among other factors. To better understand the underlying drivers of life expectancy differences between populations, demographers often employ decomposition techniques to identify the age groups and causes of death that contribute most to existing gaps. Commonly used decomposition methods in demography are the Arriaga [Arriaga, 1984], stepwise-replacement [Andreev et al., 2002], or linear integral [Horiuchi et al., 2008] decomposition algorithms.

Age-specific contributions to a given life expectancy gap are by no means uniform. A widely-studied example is the Black–white mortality crossover, by which Blacks in the United States experience higher mortality at virtually every age until around age 85 [Lariscy, 2017]. Among the age groups with higher Black mortality, infancy and young and middle adulthood have so far received the most attention in the demographic literature [Alexander and Root, Tilstra et al., 2022], leading to an improved understanding of the Black–white mortality crossover.

Because life expectancy is a nonlinear function of age-specific mortality rates, a given relative difference in mortality rates at younger ages contributes more to a life expectancy gap than if the same difference were observed at older ages, all else being equal [Aburto et al., 2020]. Although this feature of life expectancy is well known, it makes it difficult to assess whether the contribution of one age group to a given life expectancy gap is disproportionate to the contributions of all other age groups or to the gap itself. In the worst case, the nonlinearity of life expectancy can thereby lead to the interpretation of meaningful mortality differences at young ages—such as those between Black and white Americans—as mere artifacts when conventional decomposition methods are employed. Similarly, meaningful contributions at older ages may appear small if reported alongside seemingly larger contributions at younger ages.

In this abstract, we present the *Proportional Difference Comparison Method* (PDCM) to assess whether age-specific contributions to an existing life expectancy gap are (dis)proportionately large or small given that gap. Specifically, we propose to compare (a) observed age-specific contributions to a given life expectancy gap with (b) hypothetical age-specific contributions that would be observed if

the mortality rates between the two populations differed by the same relative factor at all ages, but the life expectancy gap remained fixed. Previous research has shown that a difference in mortality rates by the same factor at all ages would both increase life expectancy and reduce lifespan inequality, leading to better and more equitable mortality outcomes at the population level [Aburto et al., 2020]. This makes the proportional scenario an appropriate benchmark against which to compare observed age-specific contributions. We argue that our method can be used as both (a) an exploratory tool and (b) an evaluative tool whenever the goal of decomposition analysis goes beyond the mere description of mortality differences to infer whether or not these differences are (dis)proportionate to the observed life expectancy gap.

Our abstract is structured as follows. First, we present the theoretical foundations of our method, building on existing findings in the formal demographic literature. Second, we demonstrate our method by applying it to the United States Black–white life expectancy gap in 2019. We find that contributions to the life expectancy gap from mortality in infancy and young and middle adult were larger than would be expected under a proportional scenario. Conversely, observed contributions at older ages were smaller than the life expectancy gap suggested. This includes ages before the Black–white mortality crossover, highlighting that a mere focus on ages at which Black mortality is lower than white mortality may only provide a limited understanding of the Black–white mortality crossover.

Throughout our manuscript, we use continuous notation, assuming that both life expectancy and the underlying force of mortality change gradually along an observed or hypothetical dimension t that distinguishes two populations [Horiuchi et al., 2008] .

2 Background

Life expectancy at birth in popultation t, $e_0(t)$, is given by :

$$e_0(t) = \int_0^\omega \ell(x,t)dx,$$

where $\ell(x,t)$ is the probability to survive until exact age x in population t, and ω is the highest age observed in the population. $\ell(x,t)$ can be expressed in terms of the force of mortality, $\mu(x,t)$, as:

$$\ell(x,t) = e^{-\int_0^x \mu(a,t)da}.$$

Keyfitz [Keyfitz, 1977] showed that changing the force of mortality across all ages by a constant

factor, $\delta(t)$, results in the new probability of survival $\ell^*(x,t)$:

$$\begin{aligned}\ell^*(x,t) &= e^{-\int_0^x \delta(t)\times\mu(a,t)da} \\ &= e^{-\delta(t)\times\int_0^x \mu(a,t)da} \\ &= \left[e^{-\int_0^x \mu(a,t)da}\right]^{\delta(t)} \\ &= [\ell(x,t)]^{\delta(t)},\end{aligned} \tag{1}$$

with the new life expectancy, $e_0^*(t)$, given by:

$$e_0^*(t) = \int_0^\omega [\ell(x,t)]^{\delta(t)}\, dx = \int_0^\omega \ell^*(x,t)dx. \tag{2}$$

More generally, Vaupel and Canudas-Romo [Vaupel and Canudas-Romo, 2003] showed that the change in life expectancy resulting from age-specific improvements, $\rho(x,t)$, in the force of mortality, $\mu(x,t)$, along the dimension t is given by:

$$\frac{\partial}{\partial t}e_0 = \int_0^\omega \rho(x,t)\ell(x,t)\mu(x,t)e(x,t)dx, \tag{3}$$

where $\rho(x,t)$ is defined as the negative of the relative derivative of $\mu(x,t)$ with respect to the dimension t:

$$\rho(x,t) \equiv -\acute{\mu}(x,t) = -\frac{\frac{\partial}{\partial t}\mu(x,t)}{\mu(x,t)}. \tag{4}$$

The acute accent in equation 4 denotes the relative derivative with respect to the dimension t. If $\rho(x,t) > 0$ the force of mortality becomes more favorable. Conversely, $\rho(x,t) < 0$ indicates that the force of mortality becomes less favorable.

By rewriting and rearranging equation 3, we can find a rate of mortality improvement, $\rho^*(t)$, that is identical across all ages, but leaves the total life expectancy gap unchanged:

$$\begin{aligned}\frac{\partial}{\partial t}e_0(t) &= \int_0^\omega \rho(x,t)\ell(x,t)\mu(x,t)e(x,t)dx \\ \frac{\partial}{\partial t}e_0(t) &= \int_0^\omega \rho^*(t)\ell(x,t)\mu(x,t)e(x,t)dx \\ \frac{\partial}{\partial t}e_0(t) &= \rho^*(t) \times \int_0^\omega \ell(x,t)\mu(x,t)e(x,t)dx \\ \frac{\partial}{\partial t}e_0(t) &= \rho^*(t) \times e^\dagger(t) \\ \rho^*(t) &= \frac{\frac{\partial}{\partial t}e_0(t)}{e^\dagger(t)},\end{aligned} \tag{5}$$

where $e^{\dagger}(t)$ is a measure of lifespan inequality called *life lost* [Vaupel and Canudas-Romo, 2003]. While equations 2 and 5 describe the same case of identical improvements in the force of mortality across all ages, their notation differs. However, by using equation 4, the relationship between $\delta(t)$ and $\rho^*(t)$ can be expressed as follows:

$$\begin{aligned} 1 - \rho^*(t) &= 1 + \frac{\frac{\partial}{\partial t}\mu(x,t)}{\mu(x,t)} \\ &= \frac{\mu(x,t) + \frac{\partial}{\partial t}\mu(x,t)}{\mu(x,t)} \\ &= \delta(t). \end{aligned} \tag{6}$$

Similarly, equation 2 can be rewritten as:

$$e_0^*(t) = \int_0^{\omega} [\ell(x,t)]^{\delta(t)}\, dx = \int_0^{\omega} [\ell(x,t)]^{1-\rho^*(t)}\, dx. \tag{7}$$

Finally, based on equations 3 and 5, the relative difference in age-specific contributions to life expectancy changes between a scenario with observed rates of mortality improvement, $\rho(x,t)$, and a hypothetical scenario with a constant rate of mortality improvement, $\rho^*(t)$, is given by:

$$\frac{\rho(x,t)\ell(x,t)\mu(x,t)e(x,t) - \rho^*(t)\ell(x,t)\mu(x,t)e(x,t)}{\rho^*(x,t)\ell(x,t)\mu(x,t)e(x,t)} = \frac{\rho(x,t) - \rho^*(t)}{\rho^*(t)}, \tag{8}$$

which is equal to the relative difference between $\rho(x,t)$ and $\rho^*(t)$. We propose to express the difference between $\rho(x,t)$ and $\rho^*(t)$ as a fraction of $\rho^*(t)$ to ensure that the values in the denominator have a consistent sign. In our example of the Black–white life expectancy gap, the larger life expectancy among white Americans translates into a positive value for $\rho^*(t)$ and thereby into positive age-specific contributions in the proportional scenario. Similarly, we propose to take the difference $\rho(x,t) - \rho^*(t)$—instead of $\rho^*(t) - \rho(x,t)$—in the numerator to ensure that larger-than-expected contributions—i.e. $\rho(x,t) > \rho^*(t)$—receive a positive value, while smaller-than-expected contributions—i.e. $\rho(x,t) < \rho^*(t)$—receive a negative value.

Data & Methods

To demonstrate the *Proportional Difference Comparison Method*, we used age-specific mortality rates for non-Hispanic Blacks (hereafter "Blacks") and non-Hispanic whites (hereafter "whites") by single year of age for the year 2019 provided in the Human Life-Table Database [Max Planck Institute for Demographic Research (Germany) et al., 2024].

Using the linear integral decomposition method [Horiuchi et al., 2008], we decomposed life expectancy gaps in the following two scenarios into additive age-specific contributions: (1) $e_0^{obs,white} - e_0^{obs,Black}$ (observed scenario); (2) $e_0^{prop,white} - e_0^{obs,Black}$ (proportional scenario), where e_0 is life expectancy at birth, and *obs* and *prop* respectively denote observed and counterfactual life expectancy at birth. $e_0^{prop,white}$ is calculated based on age-specific mortality rates for Blacks adjusted by a constant factor $\delta(t)$, such that $e_0^{prop,white} = e_0^{obs,white}$, and $e_0^{prop,white} - e_0^{obs,Black} = e_0^{obs,white} - e_0^{obs,Black}$. In 2019, $e_0^{obs,white}$ and $e_0^{obs,Black}$ were respectively 76.3 and 71.3 years.

Our derivations in the Background section assume continuous mortality data. However, available mortality data are discrete and, in the case of the Human Life-Table Database, abridged into single-year age groups. While Keyfitz and Caswell [Keyfitz and Caswell, 2005] proposed an analytical solution to determine $\delta(t)$ for discrete data, this solution is not exact. To avoid biases in our decompositions from approximating $\delta(t)$ analytically, we instead follow a numerical approach to determine $\delta(t)$. Specifically, we find $\delta(t)$ by iteratively minimizing the squared difference $e_0^{obs,white} - e_0^{prop,white}$ using the `R` function `optim()` with a tolerance of 10^{-10}. In our example of decomposing the Black–white life expectancy gap in 2019, we find that $\delta(t) \approx 0.7$. Hence, $\rho^*(t) \approx 0.3$.

Following equation 8, we express differences in age-specific contributions to life expectancy gaps in the two scenarios as a percentage of age-specific contributions in the proportional scenario, i.e: $100 \times \frac{\Delta_x^{obs} - \Delta_x^{prop}}{\Delta_x^{prop}}$, where Δ_x^{obs} and Δ_x^{prop} respectively are the contributions of the mortality difference at age x in the observed and the proportional scenario to the total life expectancy gap, as taken from the linear integral decomposition.

Results

Figure 1 shows log-transformed age-specific mortality rates observed among Black (dashed line) and white (blue solid line) Americans in 2019. In addition, the red solid line shows hypothetical mortality rates among whites if the same proportional difference in mortality was observed at every age, keeping the Black–white life expectancy gap fixed.

Echoing previous research, age-specific mortality rates among whites were lower at every age until age 88, with an exception at ages 10–11. In the proportional scenario, mortality rates among whites would be lower at all ages, including above the crossover age.

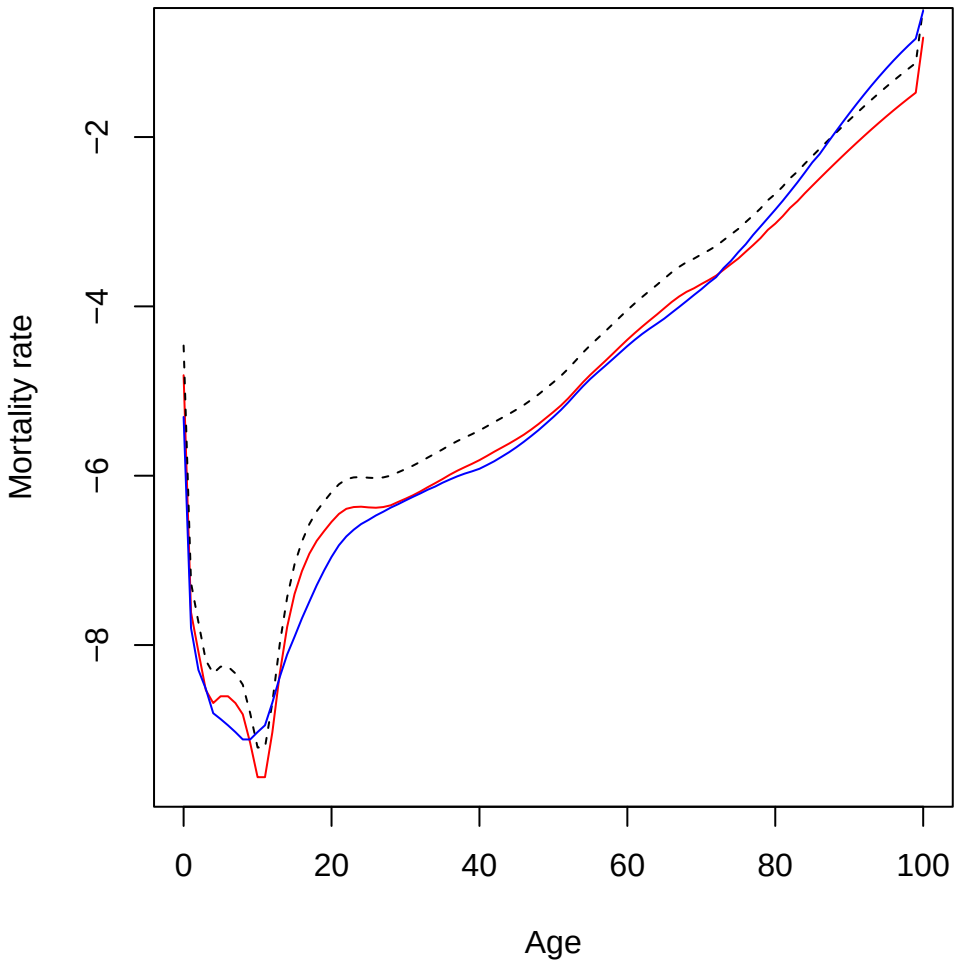

Figure 1: Age-specific log mortality rates, United States, 2019. Dashed line: observed Black mortality; blue line: observed white mortality; red line: hypothetical white mortality in proportional scenario.

Figure 2 shows age-specific contributions to the life expectancy gap between Black and white Americans in the observed scenario (black line) and the proportional scenario (red line). Reflecting the different levels of mortality in Figure 1, age-specific contributions were larger in the observed vs. proportional scenario in infancy and among all adult ages below age 73, after which contributions in the proportional scenario became larger than in the observed scenario. This age was much lower than the crossover age of 88 years, after which contributions in the observed scenario turned negative, as opposed to the positive contributions in the proportional scenario.

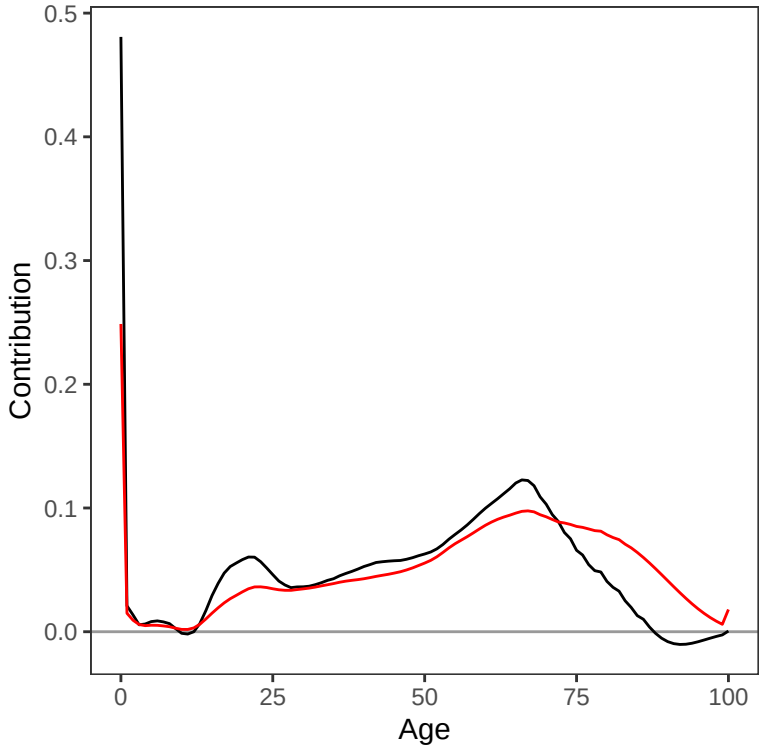

Figure 2: Age-specific contributions to life expectancy gap based on (1) observed Black and white mortality rates (black line); and (2) observed Black mortality rates and hypothetical white mortality rates in proportional scenario (red line).

Finally, Figure 3 shows the difference between the age-specific contributions in the observed and the proportional scenario, expressed relative to the contributions in the proportional scenario. Positive values indicate that the contributions in the observed scenario were larger than in the proportional scenario, whereas negative values indicate that the contributions in the observed scenario were smaller than in the proportional scenario—or, alternatively, that the contributions

were negative in the observed scenario but positive in the proportional scenario.

Three important findings stand out. First, with the exception of the ages 10–11, the observed contributions for all age groups in childhood and early and middle adulthood were larger than expected based on the proportional scenario; second, the contributions from both infant mortality and early adult mortality to the Black–white life expectancy gap exceeded expected values by roughly 100%, whereas observed contributions in middle adulthood exceed expected values by less than 50%; third, while the Black–white mortality crossover was only visible from age 88 onwards, smaller-than-expected contributions to the life expectancy gap were already visible from age 73 onwards.

3 Discussion

A central goal of demographic analysis is to contribute to a better understanding of differences in life expectancy between different populations, such as life expectancy gaps by sex, socioeconomic status, or race/ethnicity. The last decades have seen important developments in demographic decomposition methods. These new methods have allowed us to more accurately describe the age- and cause-specific contributions to life expectancy gaps between populations, thereby contributing to a better understanding of the biological and social mechanisms underlying these gaps.

In this abstract, we argue that the nonlinearity of life expectancy with respect to its underlying age-specific mortality rates necessitates the definition of a benchmark against which the age-specific contributions to life expectancy gaps can be evaluated. Since the same relative differences in age-specific mortality rates will result in larger contributions to the life expectancy gap if observed at younger versus older ages, all else being equal, a standard decomposition output may lead researchers to overstate the importance of mortality differences at younger ages, while potentially overlooking equally or more important mortality differences at older ages. The *Proportional Difference Comparison Method* proposes to compare (a) observed age-specific contributions to a given life expectancy gap with (b) hypothetical age-specific contributions that would be observed if the mortality rates between the two populations differed by the same relative factor at all ages, but the life expectancy gap remained fixed. Previous research has shown that a difference in mortality rates by the same factor at all ages would both increase life expectancy and reduce lifespan inequality, leading to better and more equitable mortality outcomes at the population level [Aburto et al., 2020]. This makes the proportional scenario an appropriate benchmark against which to compare observed

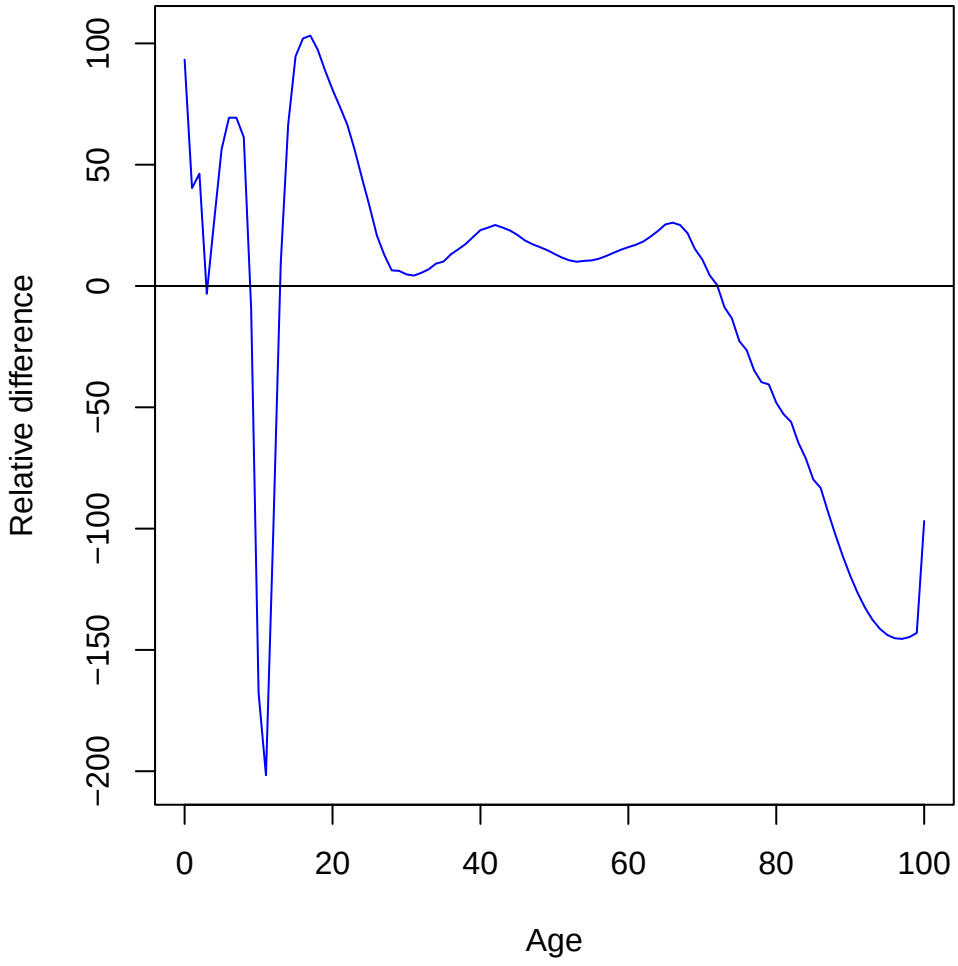

Figure 3: Relative difference between age-specific contributions to the Black–white life expectancy gap in the observed and proportional scenario.

age-specific contributions. Another advantage of this method is that the proportional factor in the hypothetical scenario can be easily derived using numerical optimization methods available in standard statistical software. This, together with the wide availability of decomposition algorithms in standard statistical software, such as `R`, makes the PDCM easy to apply with just a few lines of code.

We demonstrate our method using the example of the Black–white life expectancy gap in the United States. Existing research has described the so-called Black–white mortality crossover, according to which the mortality rates of Black Americans exceed those of white Americans until about age 85. Based on a regular decomposition, one would conclude that infant mortality is the most important factor driving the Black–white life expectancy gap. However, based on the PDCM, we find that the contributions of infant mortality to the Black–white life expectancy gap in 2019 exceeded the expected contributions based on the proportional scenario by the same factor as the contributions of young adult mortality. This suggests that both infant and young adult mortality together drive the Black–white life expectancy gap in the United States [Alexander and Root, Tilstra et al., 2022]. On the other hand, we find that at older ages, smaller-than-expected contributions become visible at age 73, much earlier than the traditional mortality crossover age. Existing research has argued that the Black–white mortality crossover may be due to selective survival and data quality issues among older Black individuals [Lariscy, 2017]. The results of our PDCM analysis suggest that some of these factors responsible for lower Black mortality may be operating well before the traditional crossover age, making the contributions to the life expectancy gap at these ages much smaller than otherwise expected. Thus, we argue that a full understanding of the Black–white mortality crossover requires a focus on a broader age range than just the ages at which Black Americans have lower mortality than white Americans.

The main limitation of our method is that the proportional scenario used to assess age-specific contributions to existing life expectancy gaps is only one of many possible benchmarks. Alternatively, one could argue that mortality improvements should be concentrated primarily at the youngest ages, as this leads to the largest improvements in life expectancy and lifespan inequality [Aburto et al., 2020]. Our proportional benchmark provides a less extreme scenario in which mortality improvements are assumed to occur proportionately at all ages. However, this benchmark could be flexibly modified depending on the normative argument being made about where mortality differences should be concentrated given a fixed life expectancy gap. We therefore encourage other researchers to develop

similar comparative methods for assessing age-specific contributions to life expectancy differences.

To summarize, the *Proportional Difference Comparison Method* presented in this abstract complements existing decomposition methods, such as the Arriaga [Arriaga, 1984], stepwise-replacement [Andreev et al., 2002], or linear integral [Horiuchi et al., 2008] decomposition algorithms, to assess whether age-specific contributions to a given life expectancy gap are smaller or larger than expected given that gap. We are optimistic that PDCM may be useful for assessing age-specific contributions to life expectancy gaps in various contexts, such as sex or socioeconomic differences in life expectancy or cross-country analysis of life expectancy gaps. Thus, our method may help to further elucidate the biological and social pathways that are responsible for shaping age-specific contributions to life expectancy differences between populations.

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